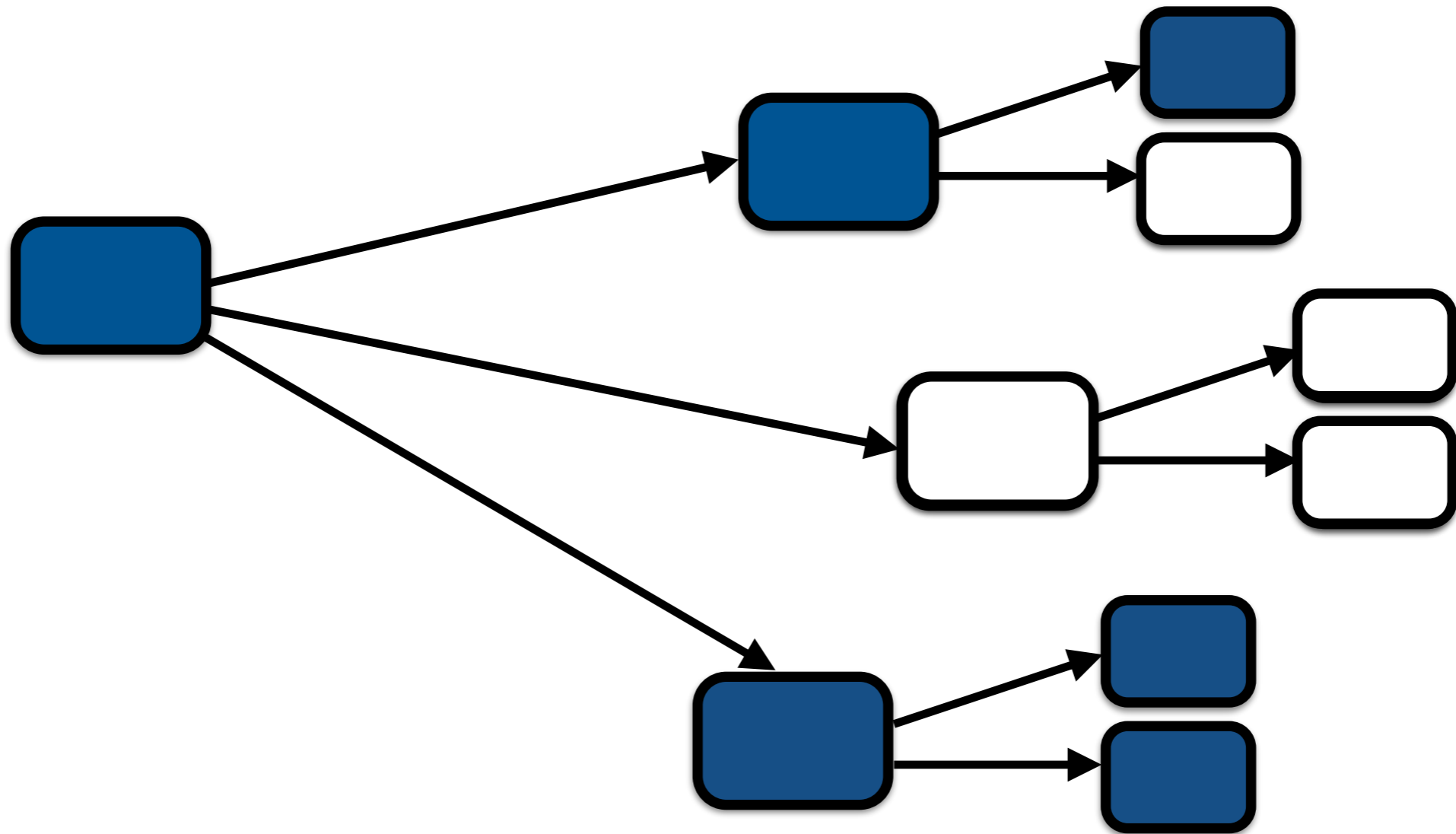


# Diffusion on a Network (Part 1): Epidemics

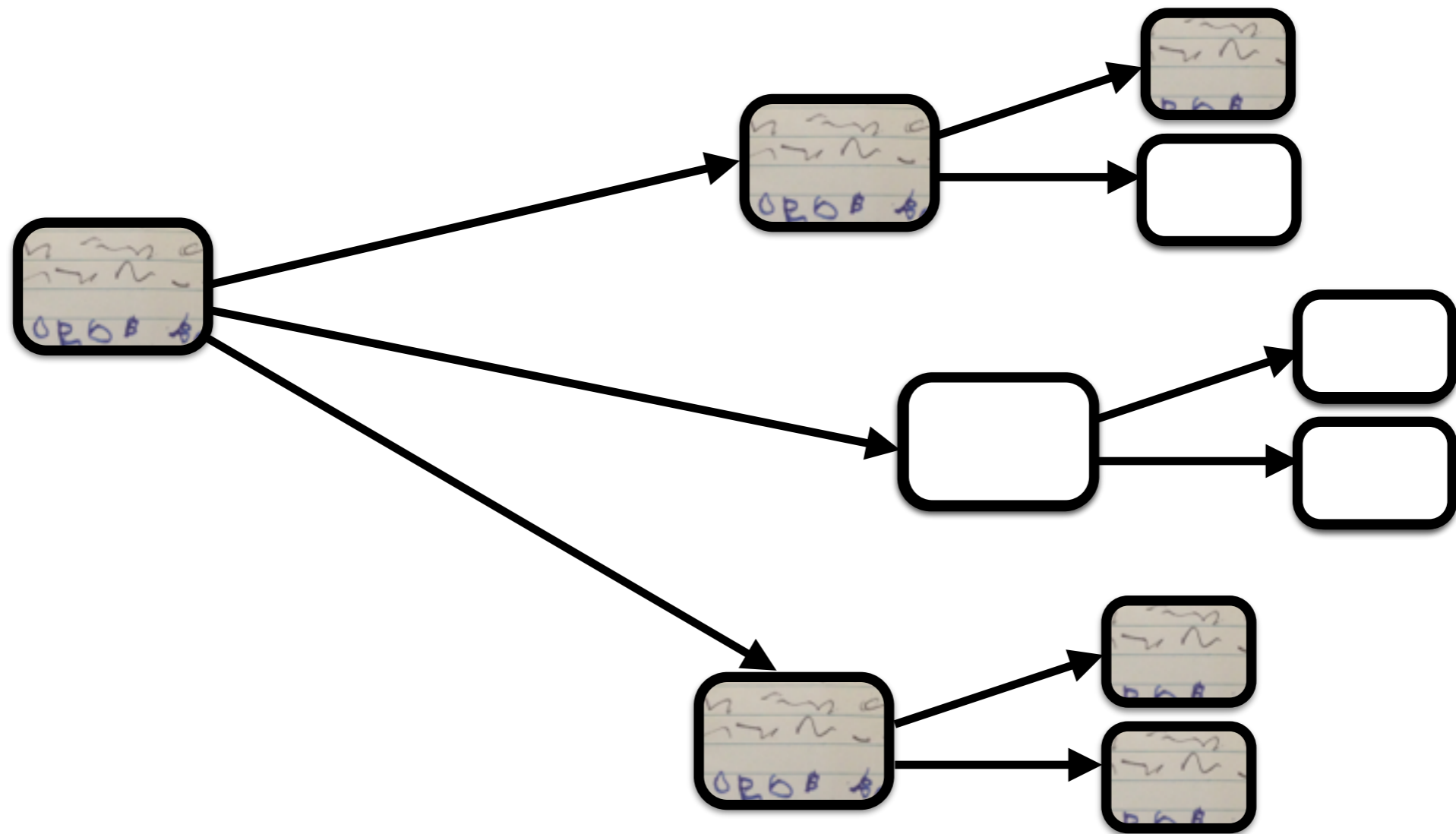
# Diffusion

When we talk about things spreading through a population, we call it *diffusion*



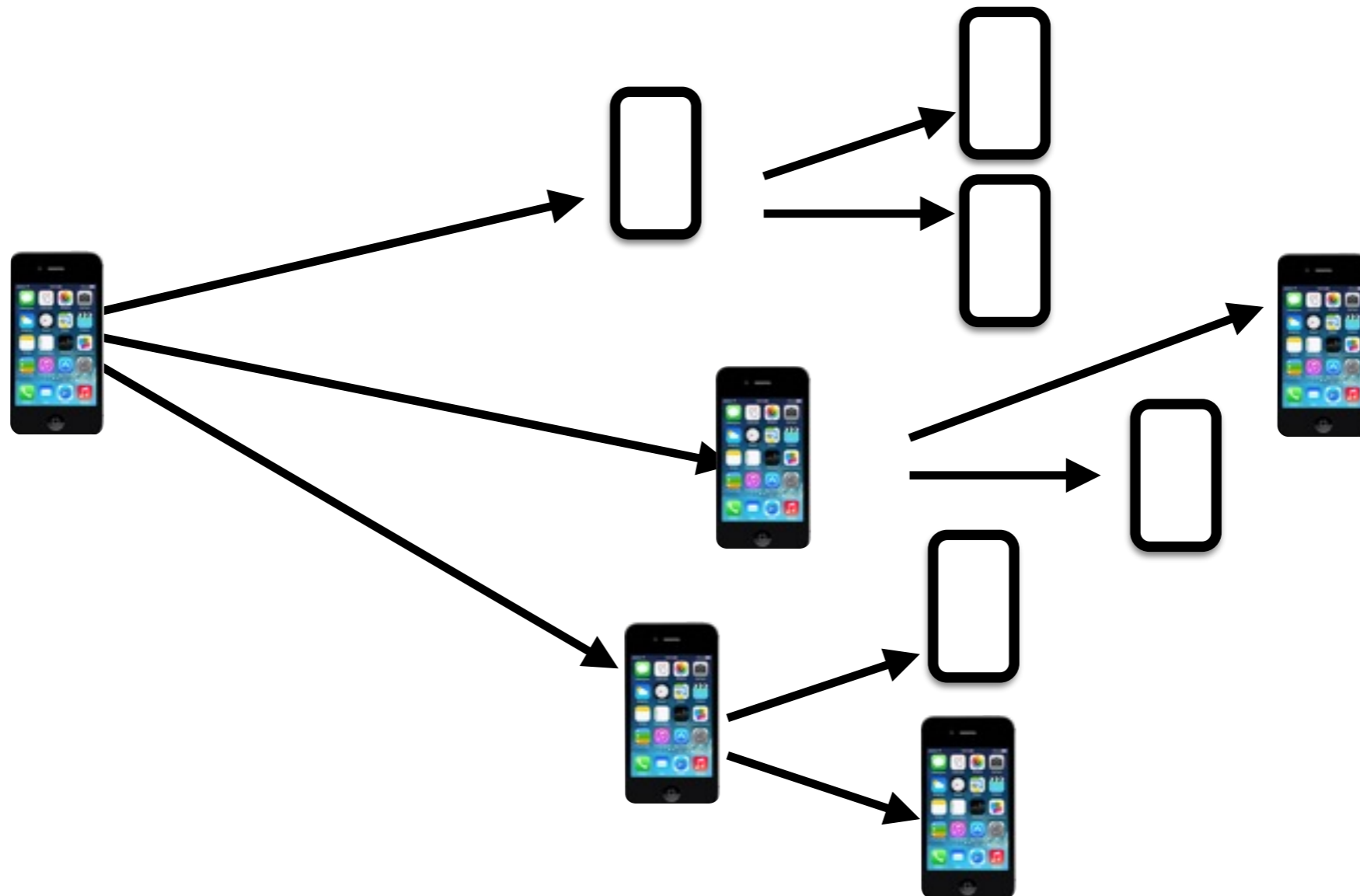
# Diffusion

There are many different things that can diffuse over a social network...information



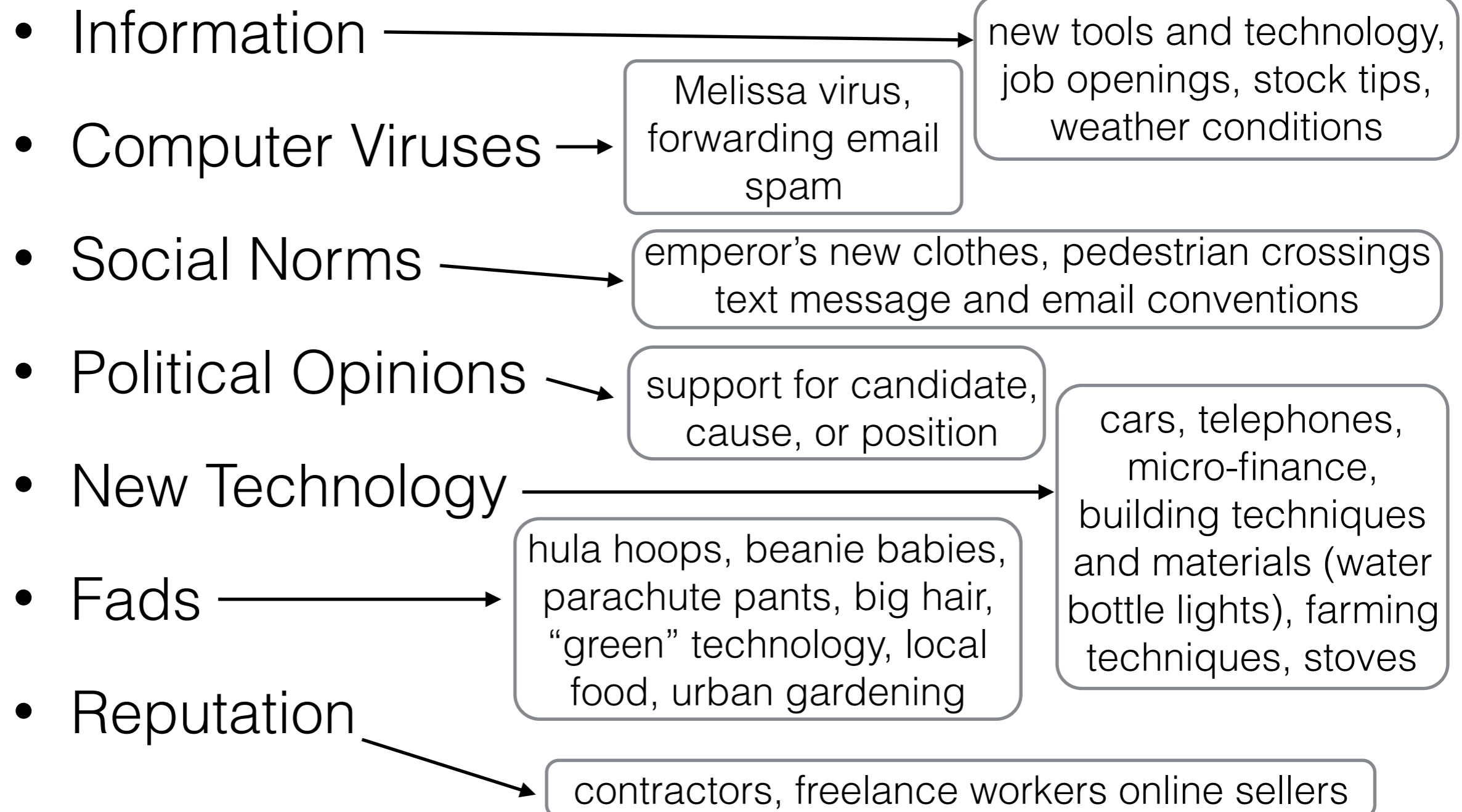
# Diffusion

...technology...



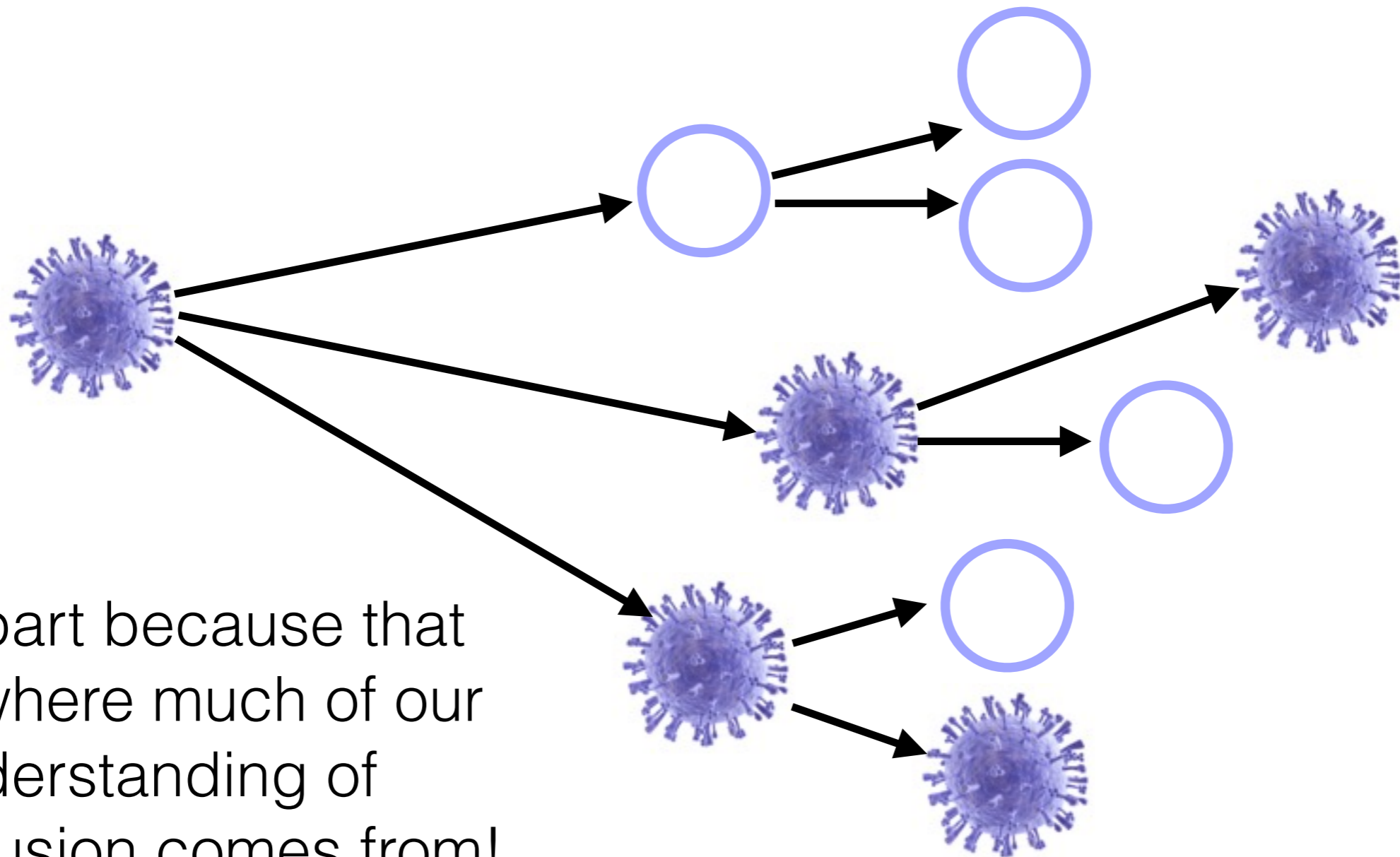
# Diffusion

And lots of other things...



# Diffusion

Today, we are going to talk about diseases...



In part because that is where much of our understanding of diffusion comes from!

# Diseases: Epidemiology

Epidemiology: The study of the spread (and control) of disease

What epidemiologists want to know:

1. How is a given disease likely to spread in a given population, without any intervention?
2. What are the best methods to control the spread of the disease?

# Diseases: Epidemiology

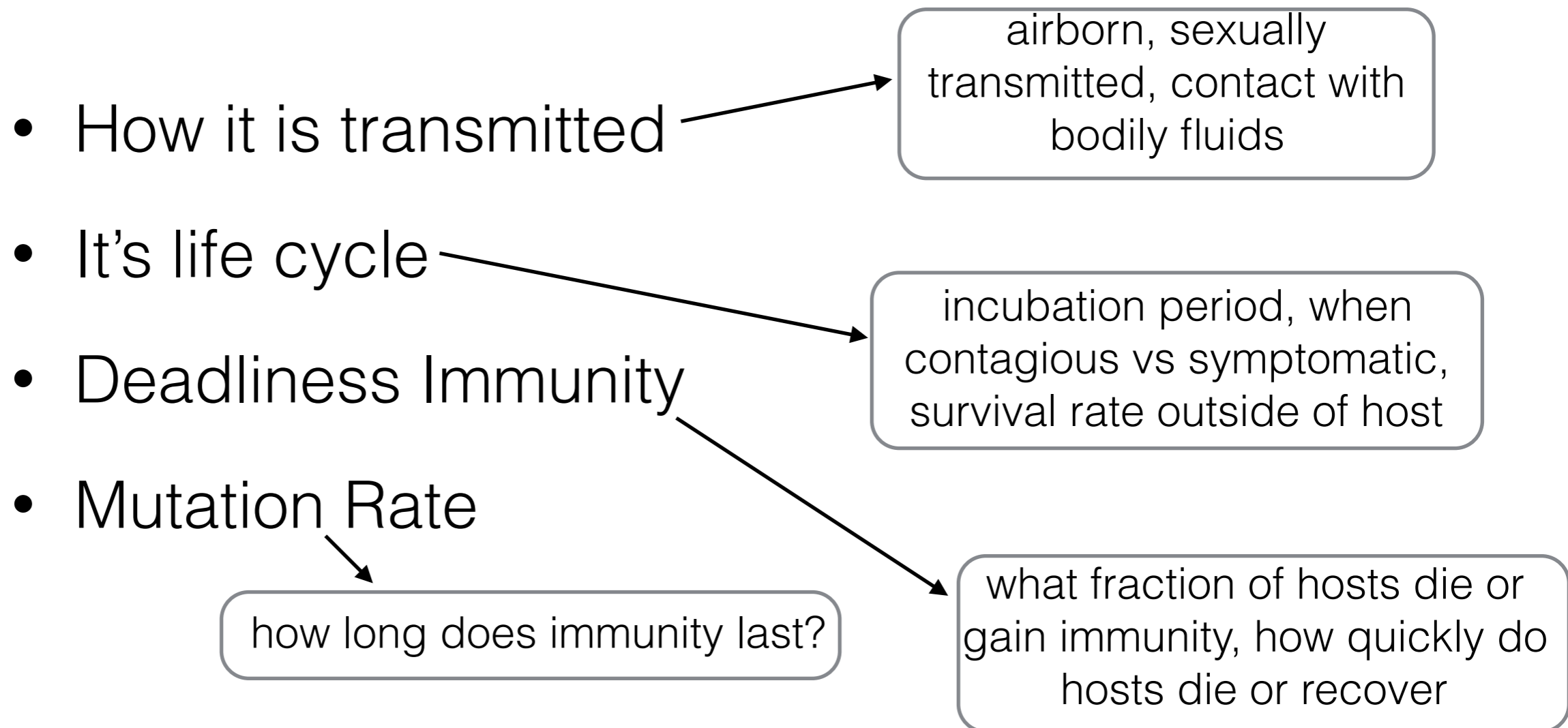
Oversimplifying (more than a little), how a disease spreads in a particular population depends on two things:

1. Characteristics of the disease (mutation rate, deadliness, transmission rate)
2. Characteristics of the network the disease spreads on (average degree, average distance, degree distribution)



# Diseases: Epidemiology

What characteristics of the disease affect it's spread?



# Diseases: Epidemiology

Contrasting different viruses:

Airborn

Doesn't live long on surfaces

**Influenza:** Host is contagious even when not symptomatic

Not particularly deadly

Hosts are mobile while infectious

Mutates quickly

Blood-born

Infectiousness depends on type of exposure

**HIV:** Host is contagious even when not symptomatic

Used to be very fairly deadly, but not any more

Very slow to mutate

Requires contact with bodily fluids

Extremely infectious, given contact

**Ebola:** Can live for a long time on surfaces

Extremely deadly

Host can remain contagious after recovery

Slow to mutate

# Diseases: Epidemiology

Contrasting different viruses:

**Influenza:** airborne + host contagious without knowing it + not deadly  
+ quick to mutate = frequent, widespread outbreaks,  
difficult to control

**HIV:** blood-born + host contagious without knowing it +  
not deadly + slow to mutate = size of outbreaks  
depends on external factors (eg: awareness),  
possible to control

**Ebola:** direct contact + not contagious unless symptomatic +  
very deadly + slow to mutate = infrequent outbreaks, tend  
to be easily controlled, but crops up again and again

# Diseases: Epidemiology

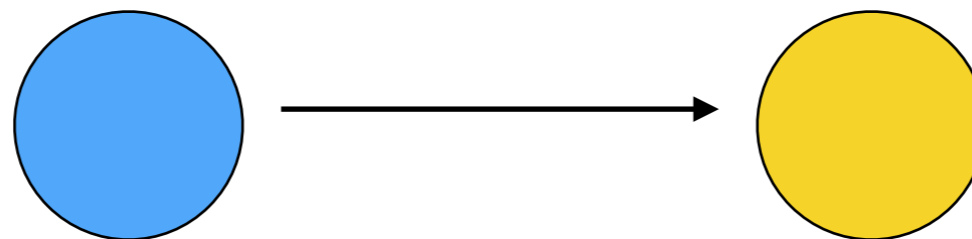
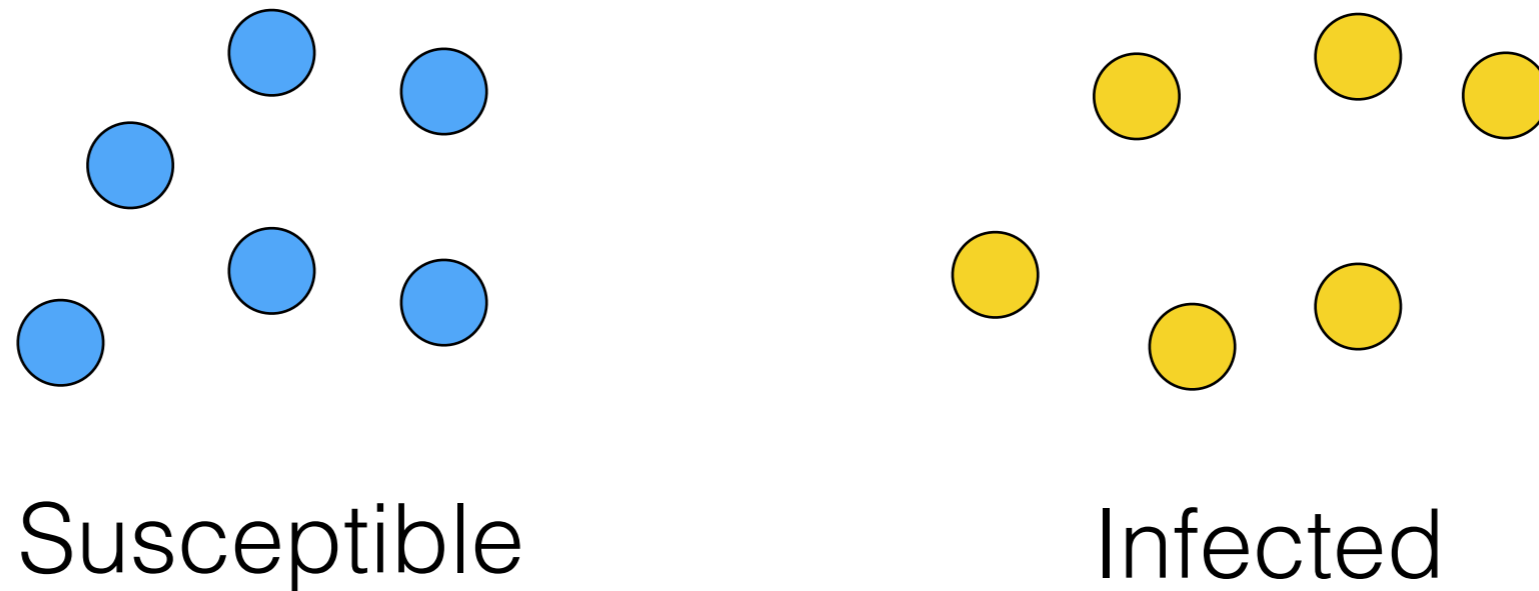
Suppose we are modeling the spread of flu on campus:

- To ignore the network, assume that every day you interact with a random group of people.
- Each day, you have the same probability of interacting with any other person in the university
- This is called “perfect mixing”

It is the basis of a classic model of disease spread: the *SI model*

# Diseases: The SI Model

People fall into one of two groups:



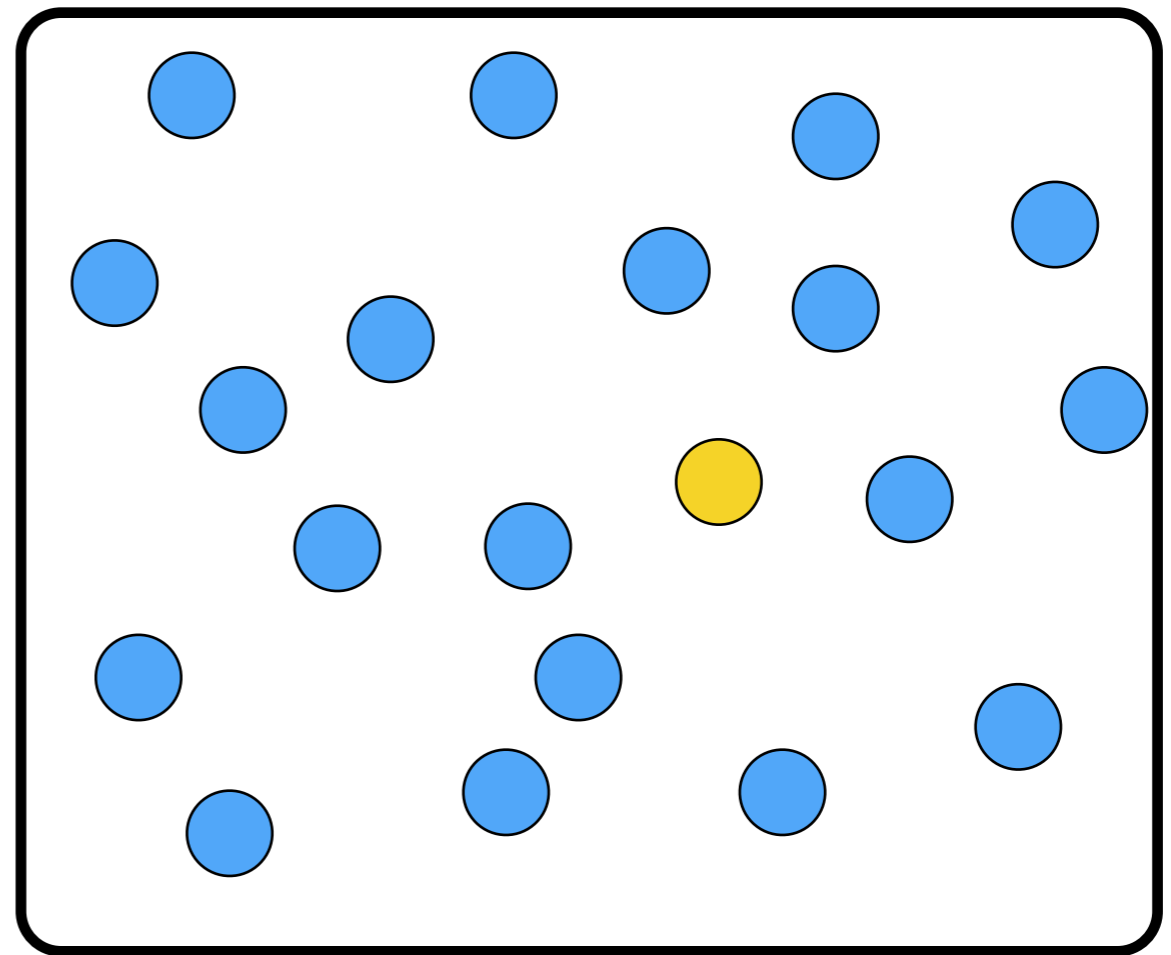
People can move from being susceptible to being infected

# Diseases: The SI Model

Population of  $N$  people

Initially,  $I_0$  people are infected

The rest are susceptible to infection:  $S_0 = (N - I_0)$



$$N = 20$$

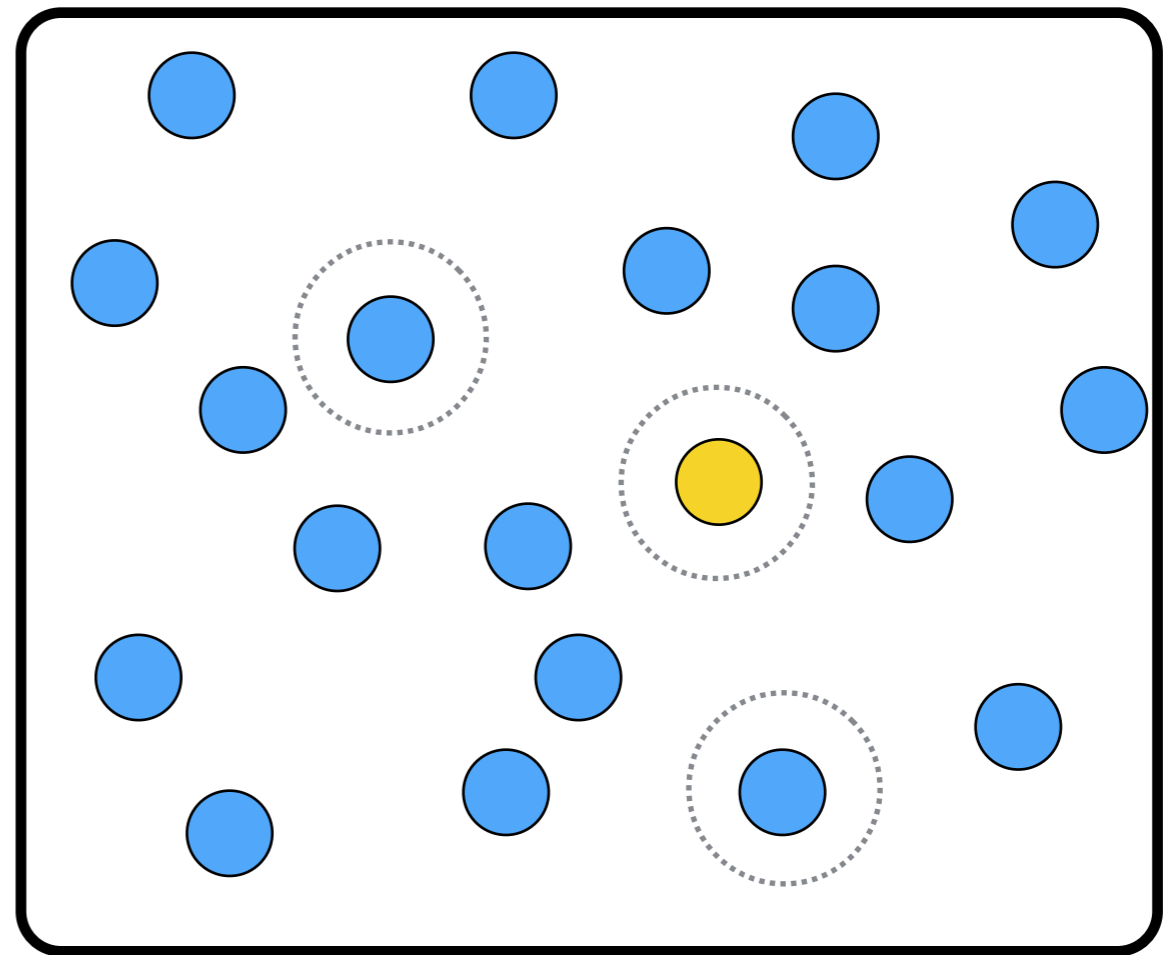
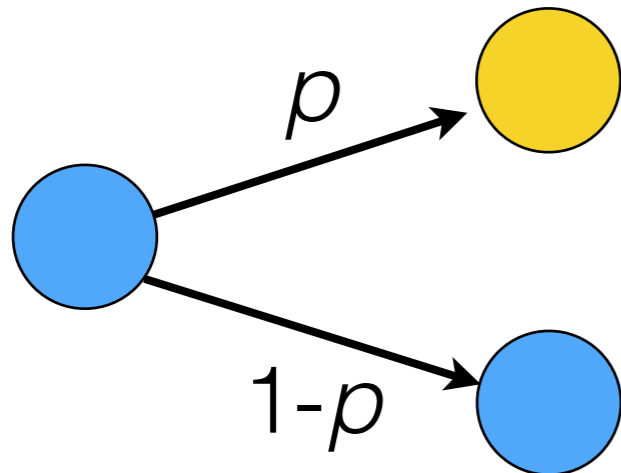
$$I_0 = 1$$

$$S_0 = 19$$

# Diseases: The SI Model

In each time period, you encounter  $c$  other people at random

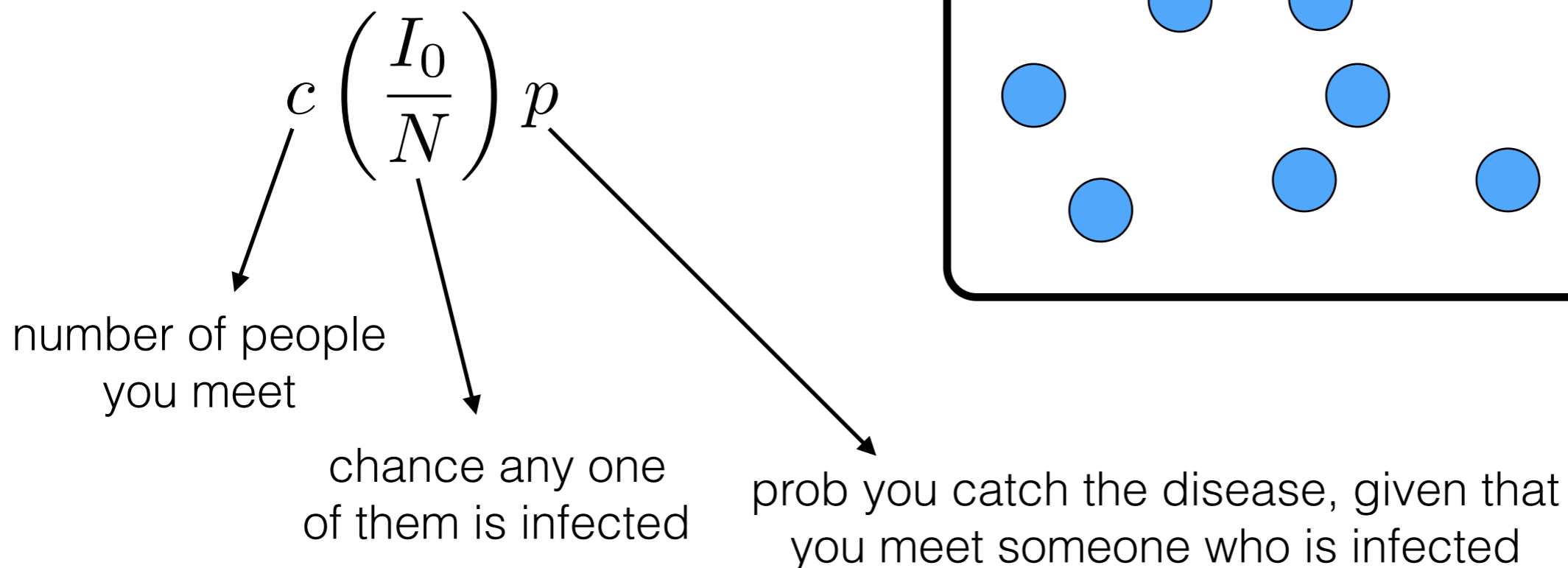
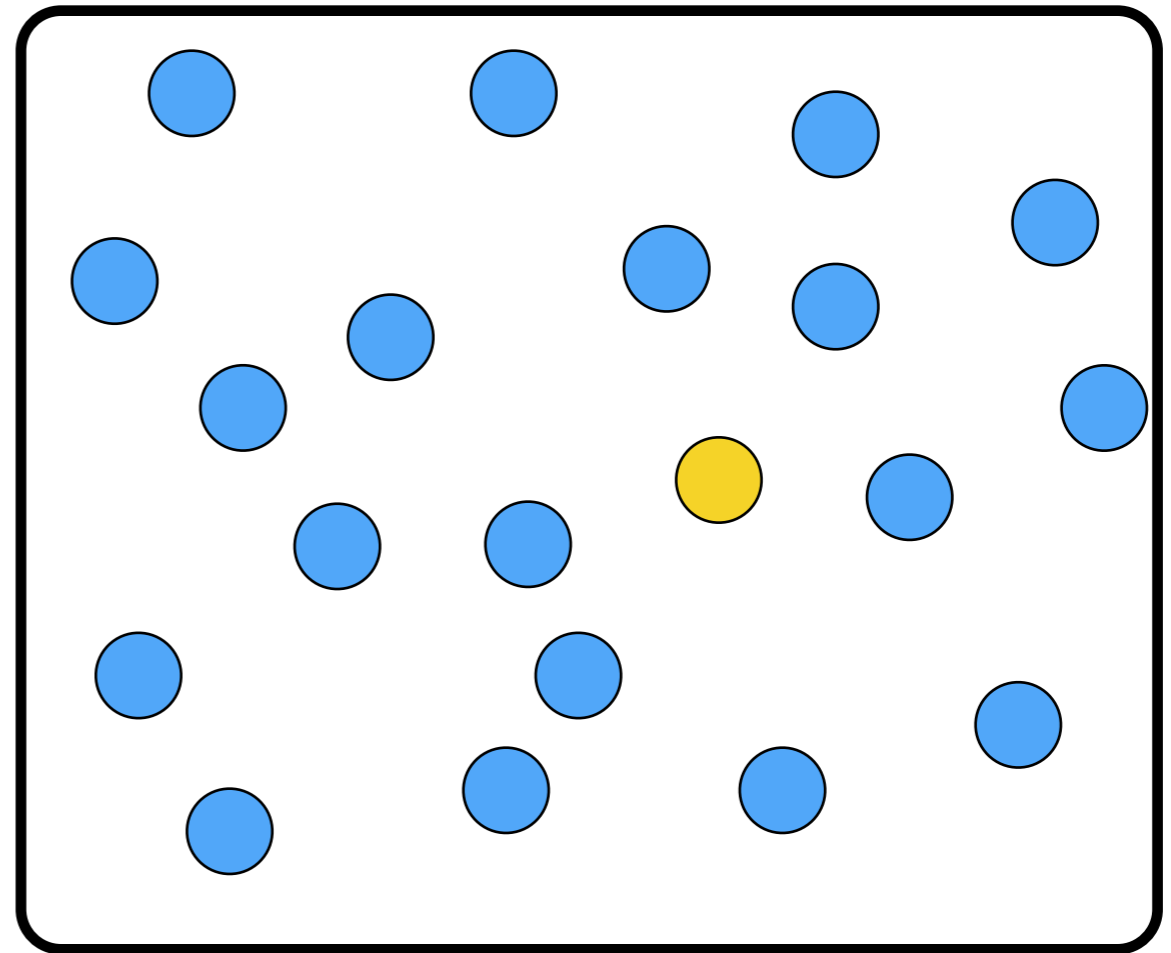
For each infected person you encounter, you become infected with probability  $p$



With probability  $(1-p)$ , you aren't infected, and remain susceptible

# Diseases: The SI Model

In the first period, the probability that you get infected = prob(meet an infected individual) \* prob(they infect you)



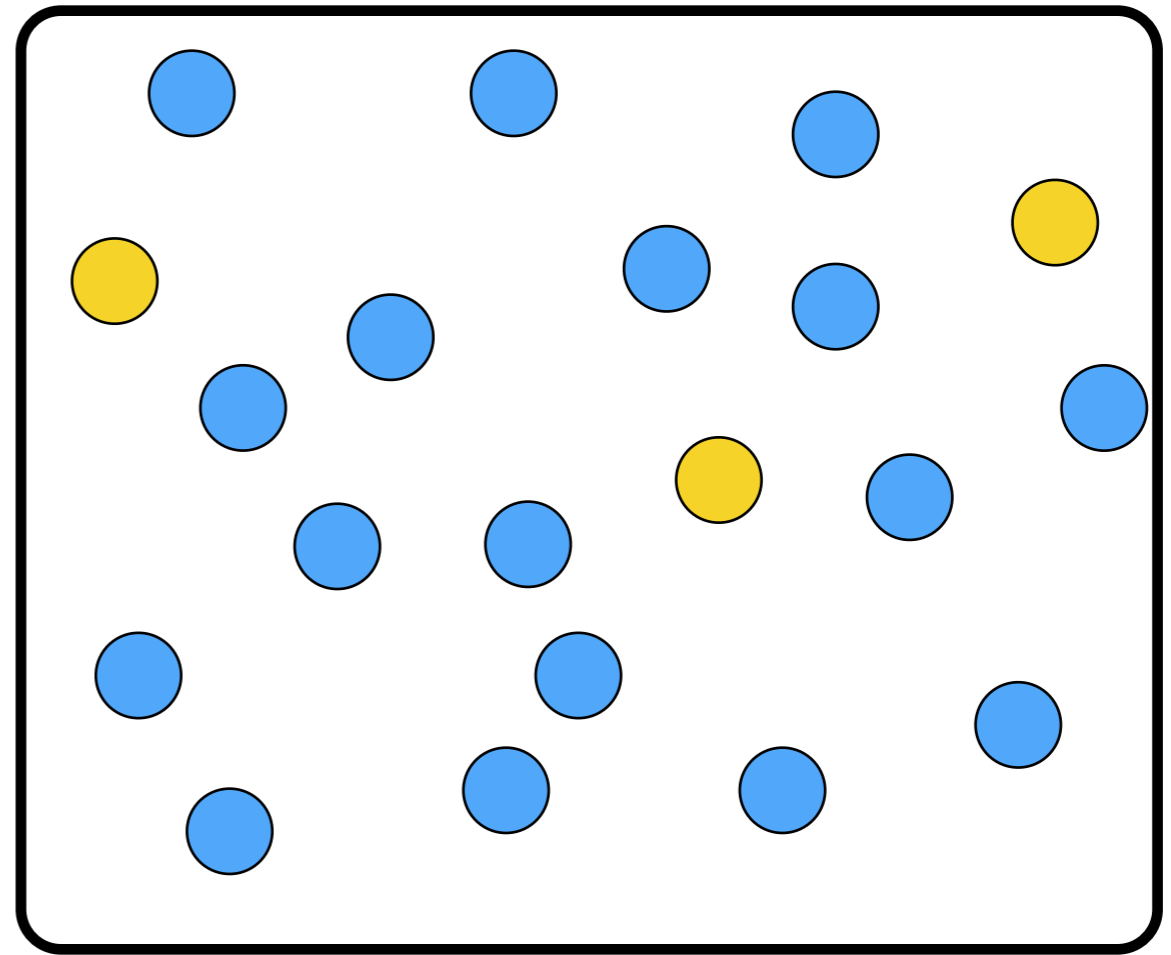


# Diseases: The SI Model

So the total number of people infected after the first period is:

$$I_1 = S_0 \underbrace{c \left( \frac{I_0}{N} \right) p}_{\text{chance any one of them is infected}} + I_0$$

$S_0$  → number of people who could be infected  
 $I_0$  → number already infected

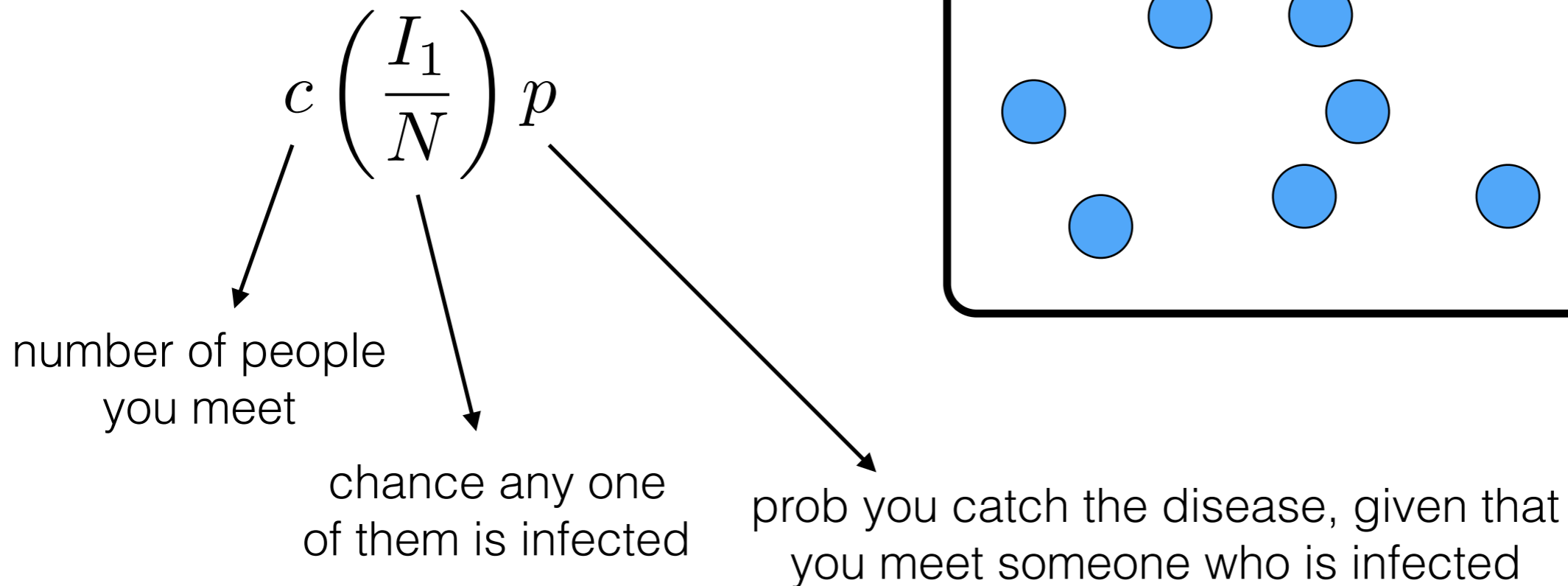
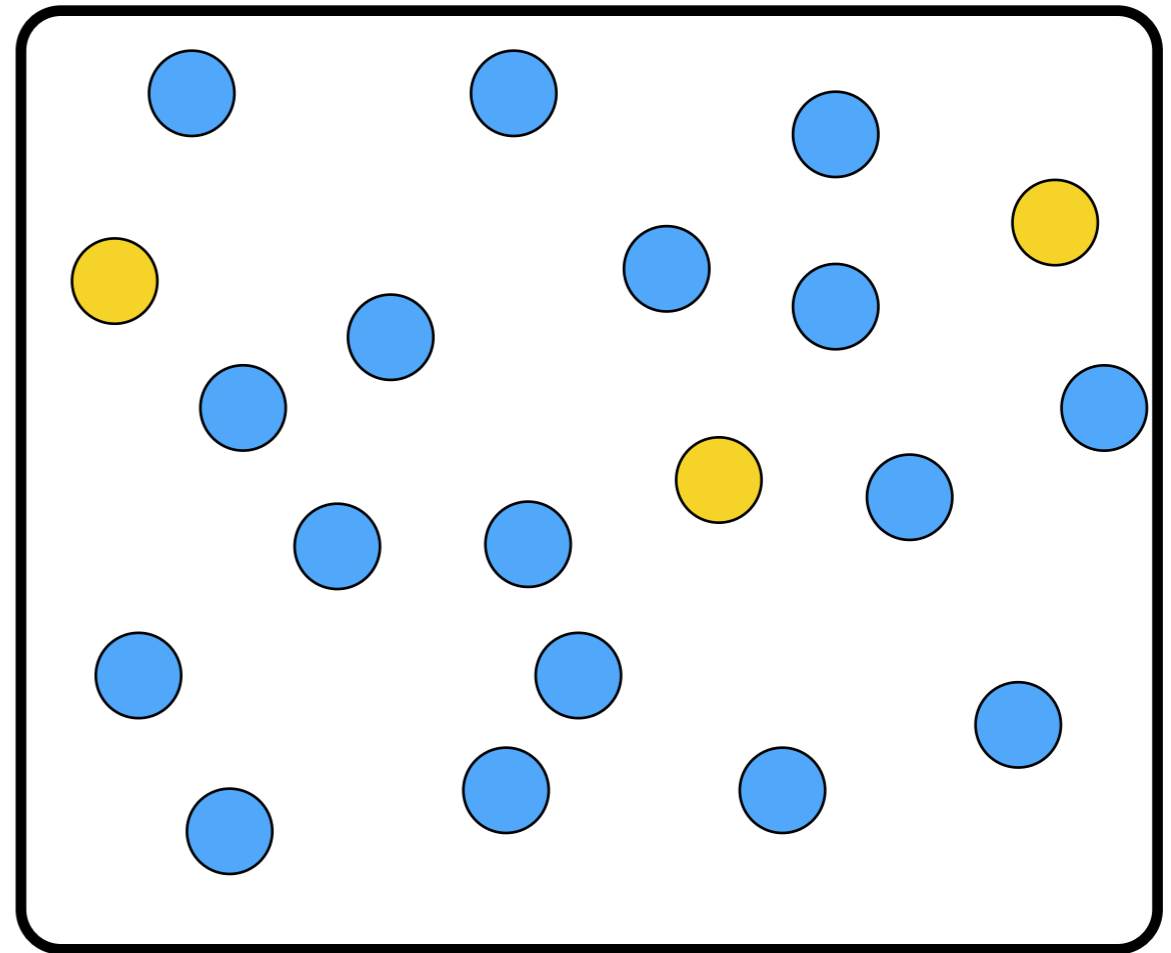


So if  $c = 3$  and  $p = .66$ , then

$$I_1 = 19 \left( 3 \left( \frac{1}{20} \right) \left( \frac{2}{3} \right) \right) + 1 = 2.9$$

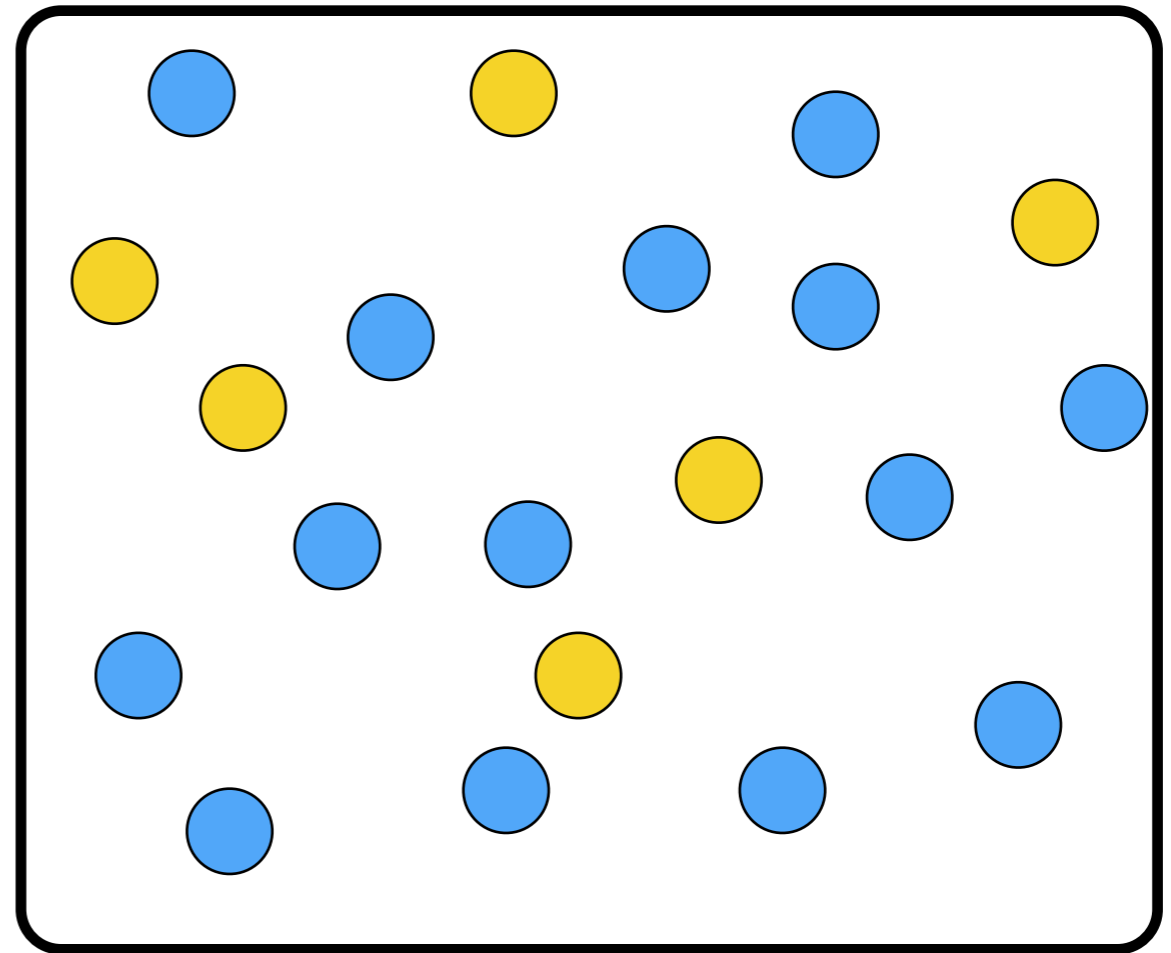
# Diseases: The SI Model

Now repeat...the chance you get infected in the second period is



# Diseases: The SI Model

So the total number of people infected after the second period is:



$$I_2 = S_1 c \left( \frac{I_1}{N} \right) p + I_1$$

number of people who could be infected      chance any one of them is infected      number already infected

So if  $c = 3$  and  $p = .66$ , then

$$I_2 = 17.1 \left( 3 \left( \frac{2.9}{20} \right) \left( \frac{2}{3} \right) \right) + 2.9 = 5.959$$

# Diseases: The SI Model

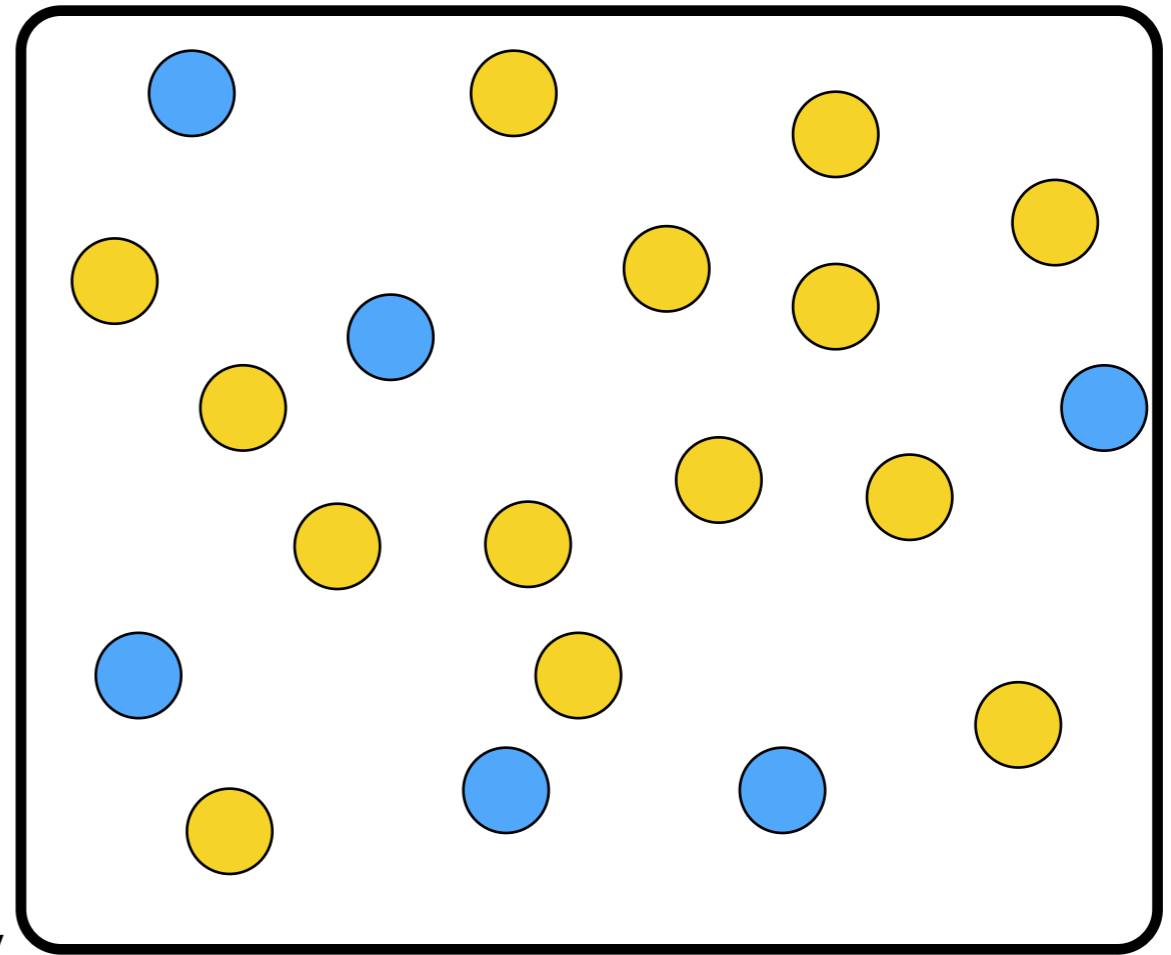
And by extending the pattern, the number infected after  $t$  periods is

$$I_t = S_{t-1} c \left( \frac{I_{t-1}}{N} \right) p + I_{t-1}$$

number of people susceptible after the last period

chance any one of them is infected

number already infected



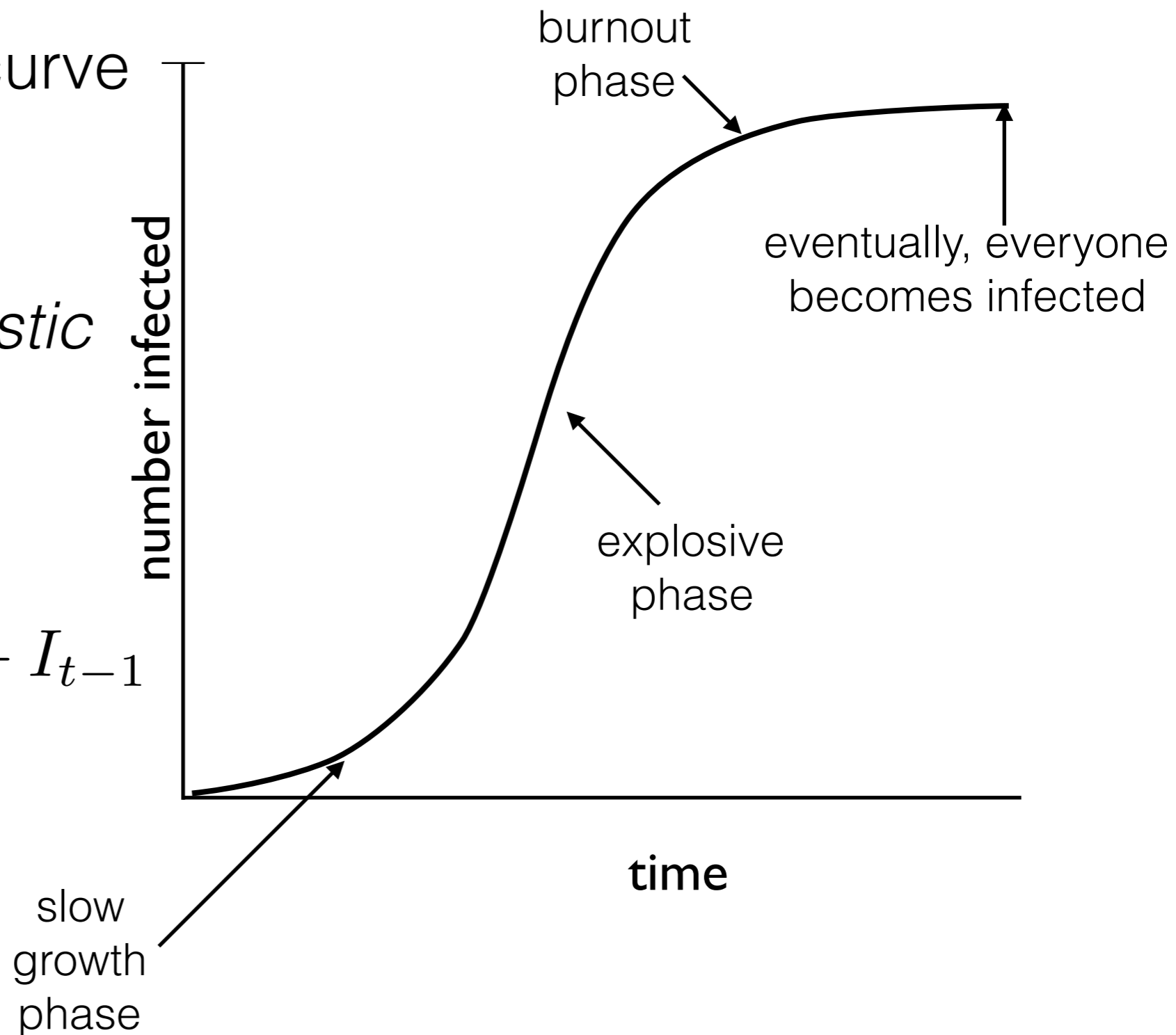
So if  $c = 3$  and  $p = .66$ , then  
 $I_3 \approx 14$

# Diseases: The SI Model

Result is a growth curve that looks like this:

This is called a *logistic* growth curve

$$I_t = S_{t-1}c \left( \frac{I_{t-1}}{N} \right) p + I_{t-1}$$



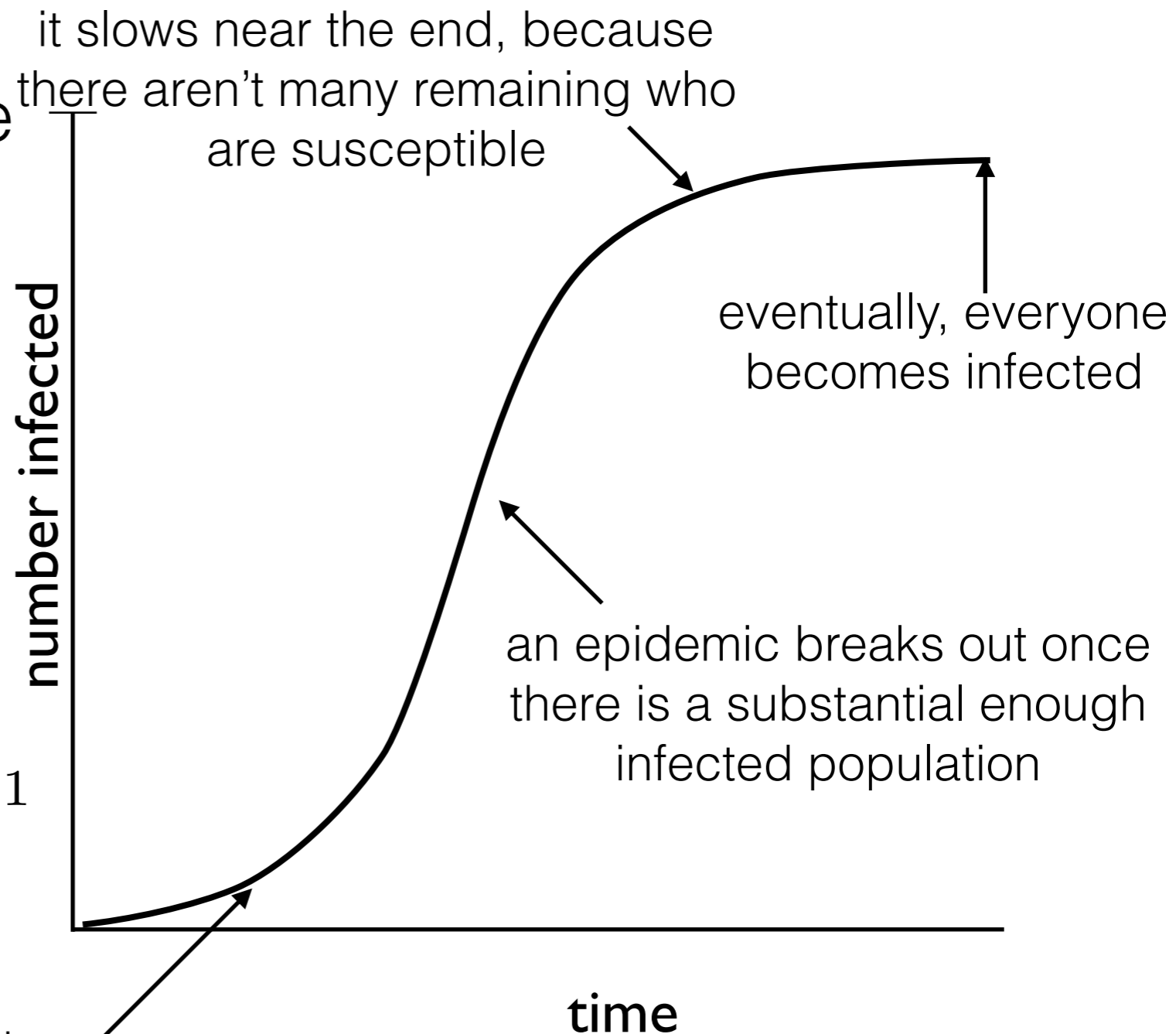
# Diseases: The SI Model

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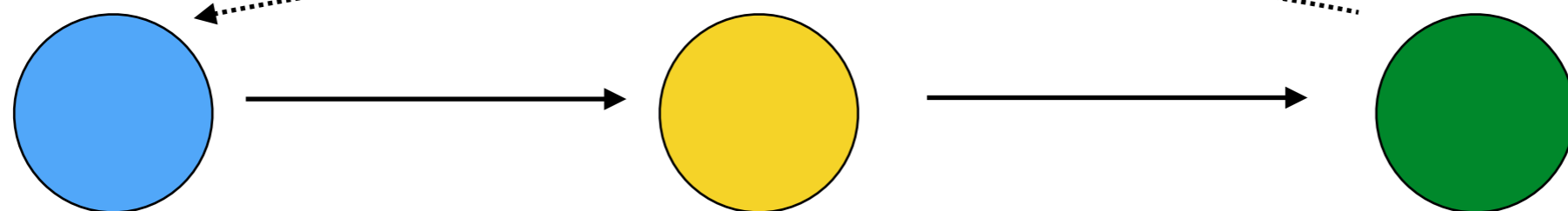
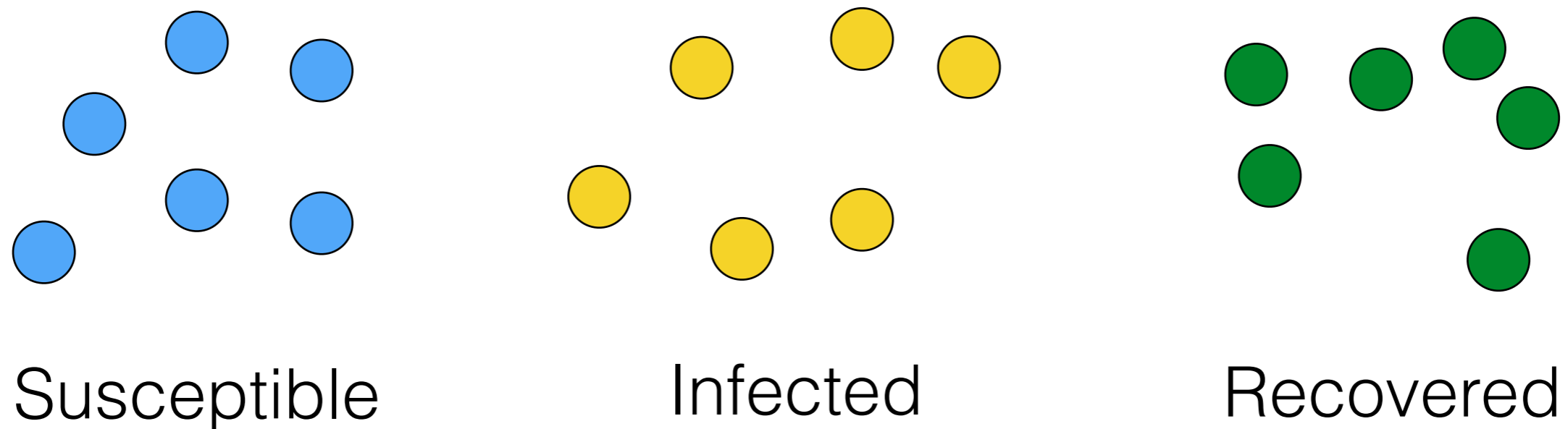
$$I_t = S_{t-1} c \left( \frac{I_{t-1}}{N} \right) p + I_{t-1}$$

initially, the disease spreads slowly, because there aren't many people infected



# Diseases: The SIR Model

The SIR model modifies the SI model: now there are three groups



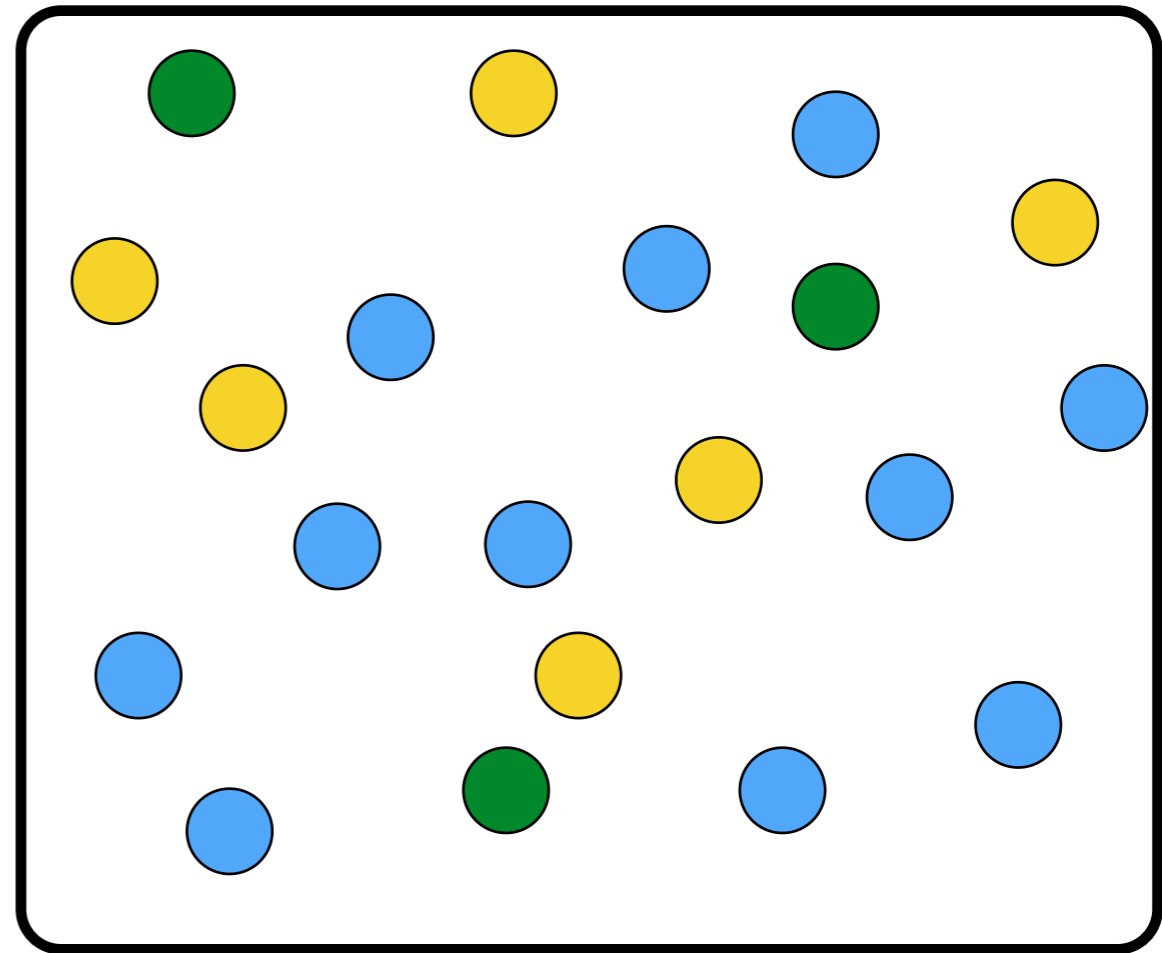
note: sometimes  
"recovery"  
actually means  
death

People move from susceptible to infected to recovered, and sometimes back to susceptible

# Diseases: The SIR Model

Now, people recover with probability  $r$

People who recover are not susceptible to the disease, and they also can't infect anyone



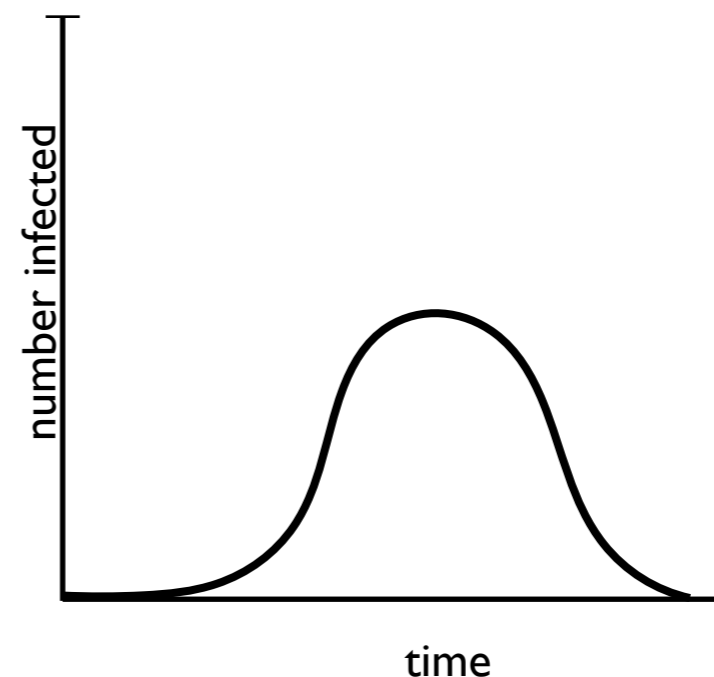
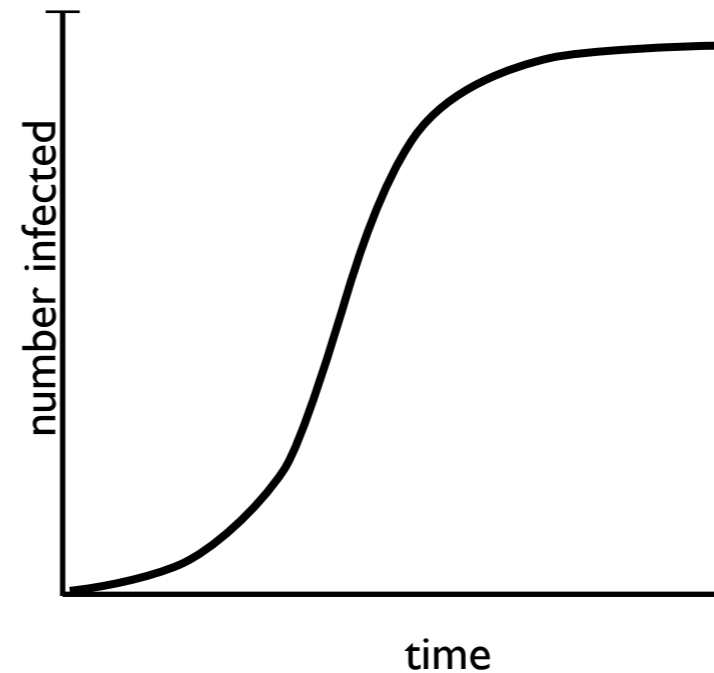


# Diseases: The SIR Model

In the SI model, there was only one possible outcome (everyone is infected).

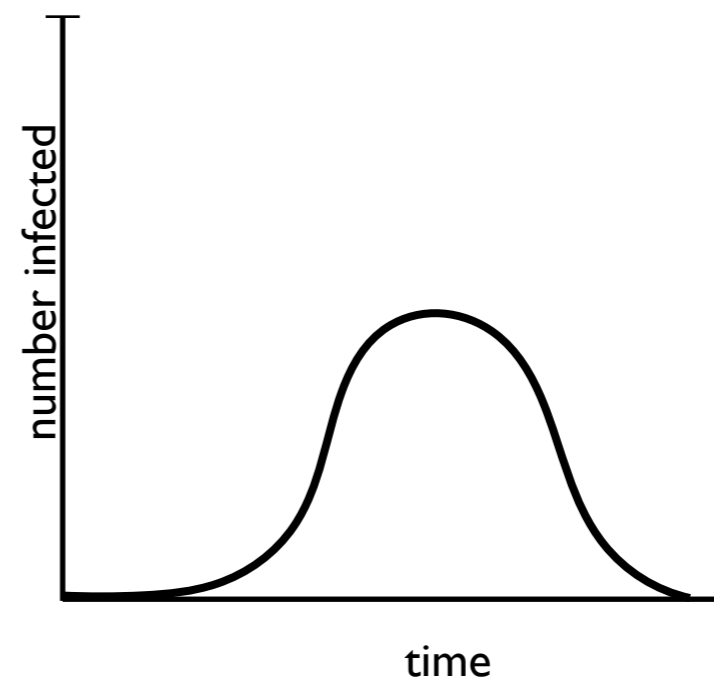
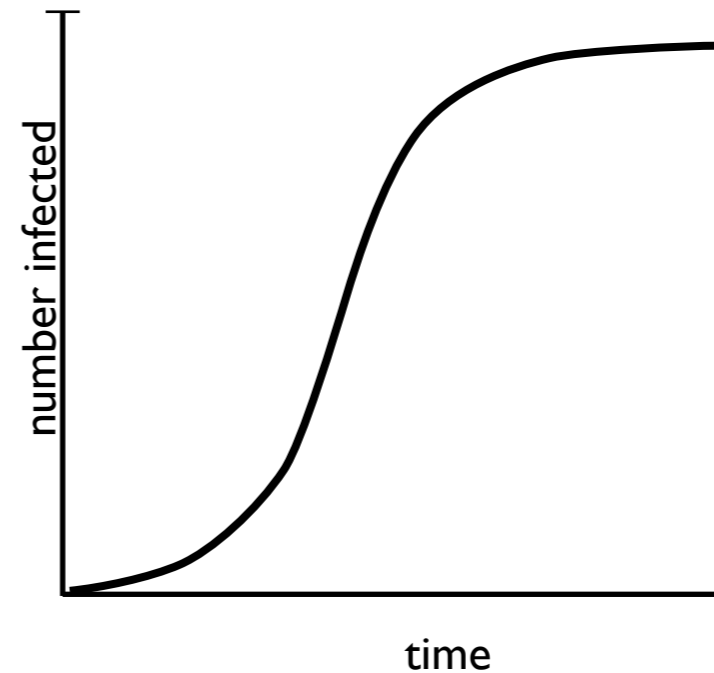
Now there are several possible outcomes:

- 1) An epidemic (everyone becomes infected)
- 2) The disease dies out before infecting everyone



# Diseases: The SIR Model

Whether the outbreak turns into an epidemic depends on the balance between the recovery rate ( $r$ ) and the infection rate ( $p$ )



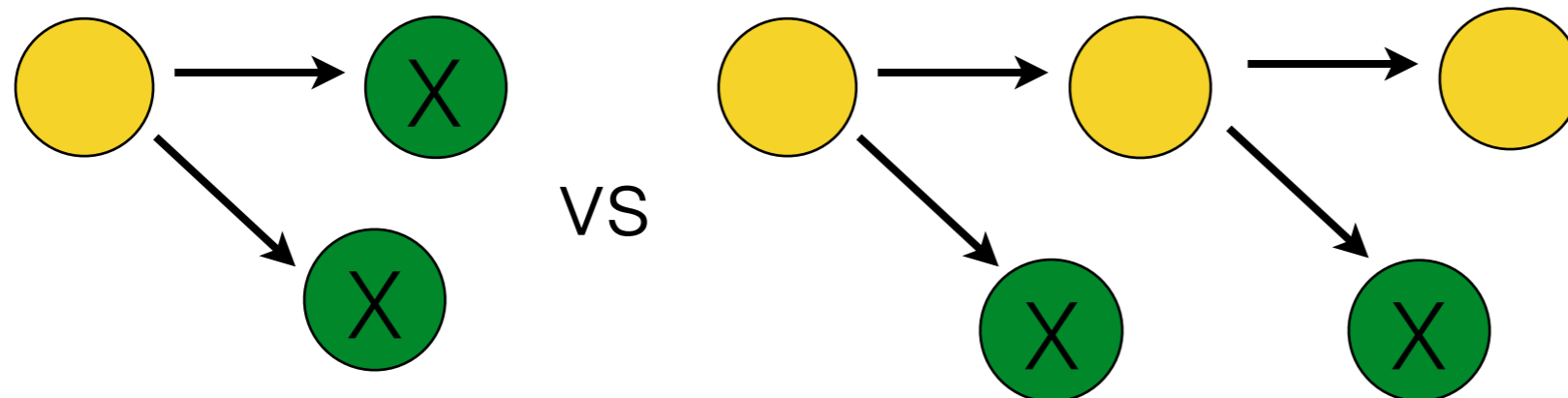
# Diseases: Epidemiology

Characteristics of a disease that affect the probability of an epidemic:

deadlier diseases kill people before the disease can spread

low mutation rates mean that recovered people remain immune

low transmission rates mean that you're less likely to get the disease from incidental contact



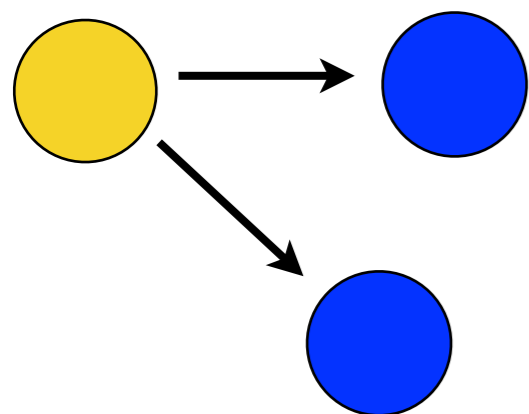
# Diseases: Epidemiology

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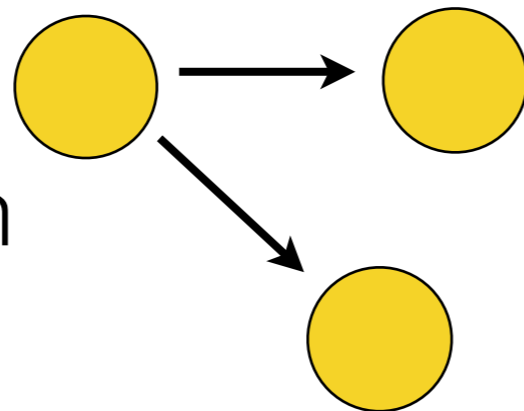
deadlier diseases kill people before the disease can spread

low mutation rates mean that recovered people remain immune

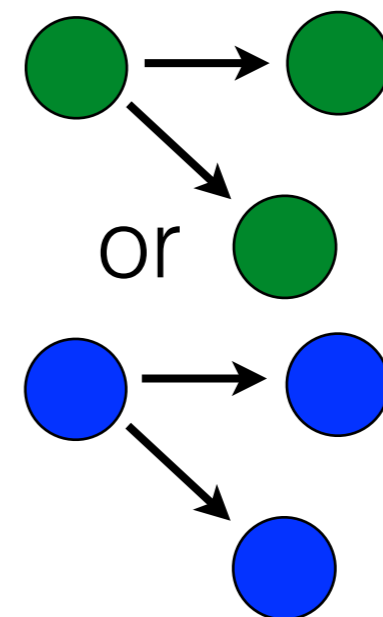
low transmission rates mean that you're less likely to get the disease from incidental contact



then



then



low mutation

or

high mutation

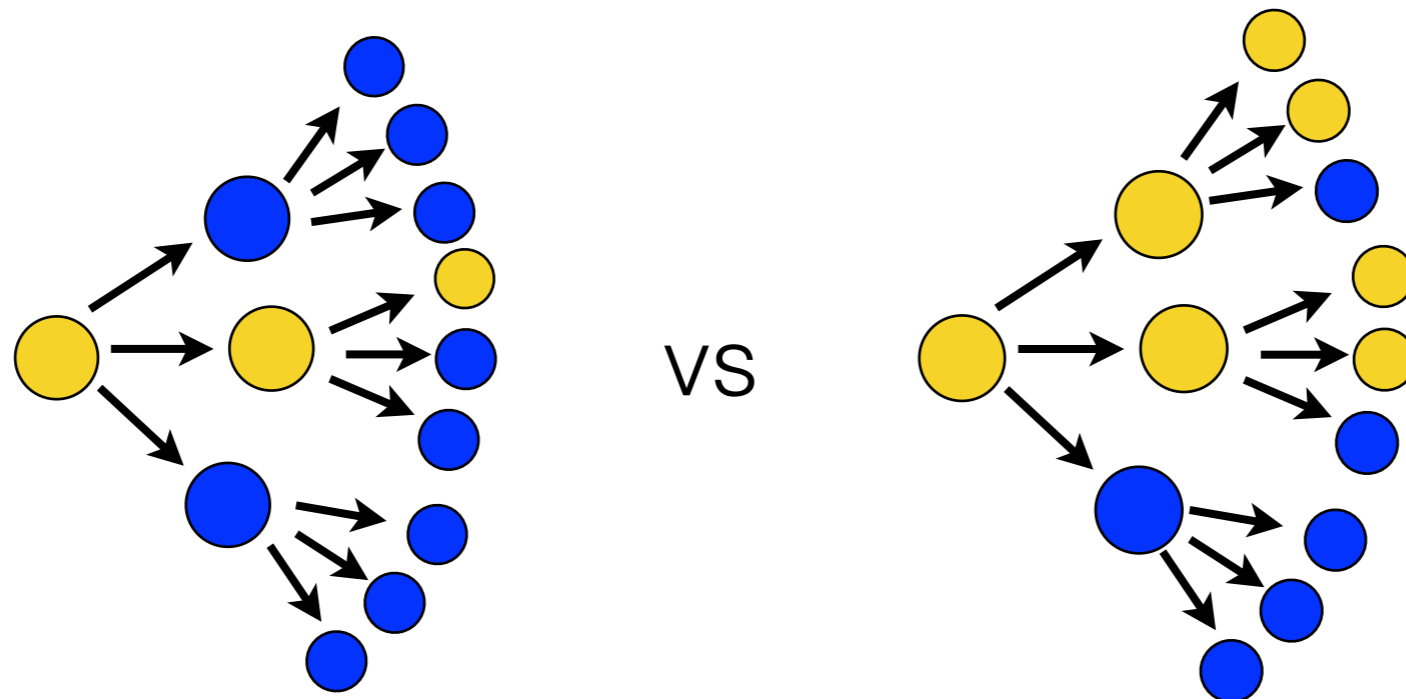
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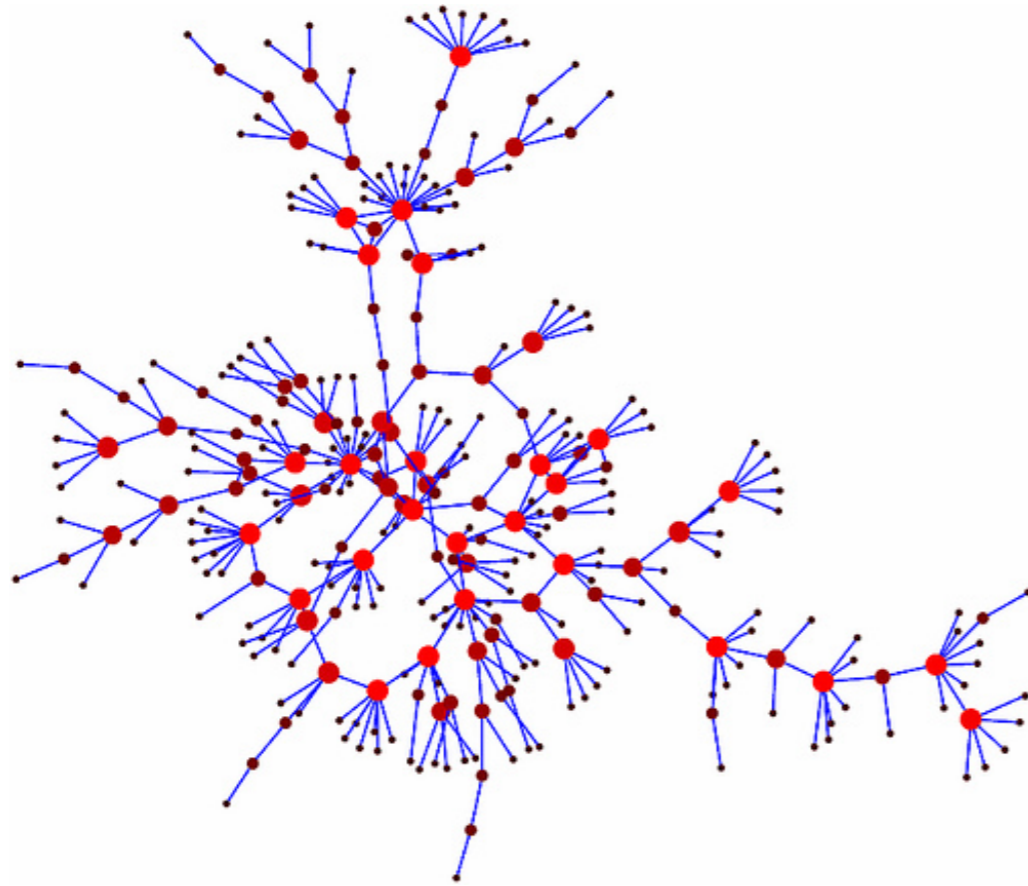
low mutation rates mean that recovered people remain immune

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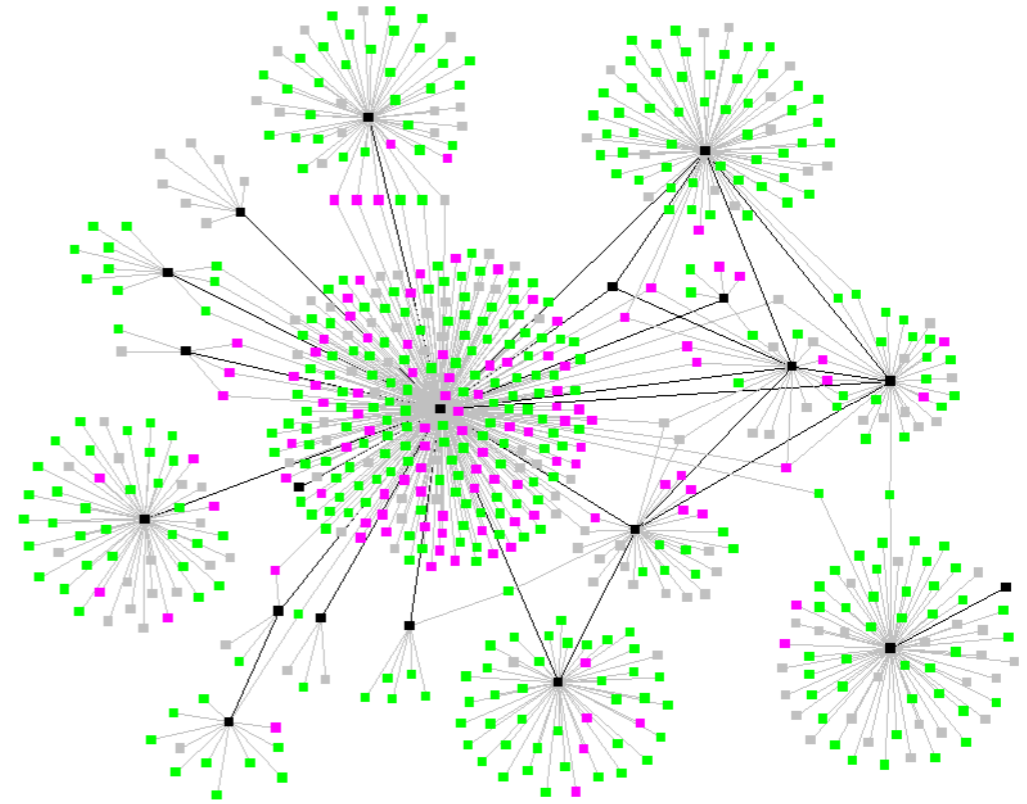
# Diseases: The role of the network

So now, what if people don't interact at random?



Sexual Contacts

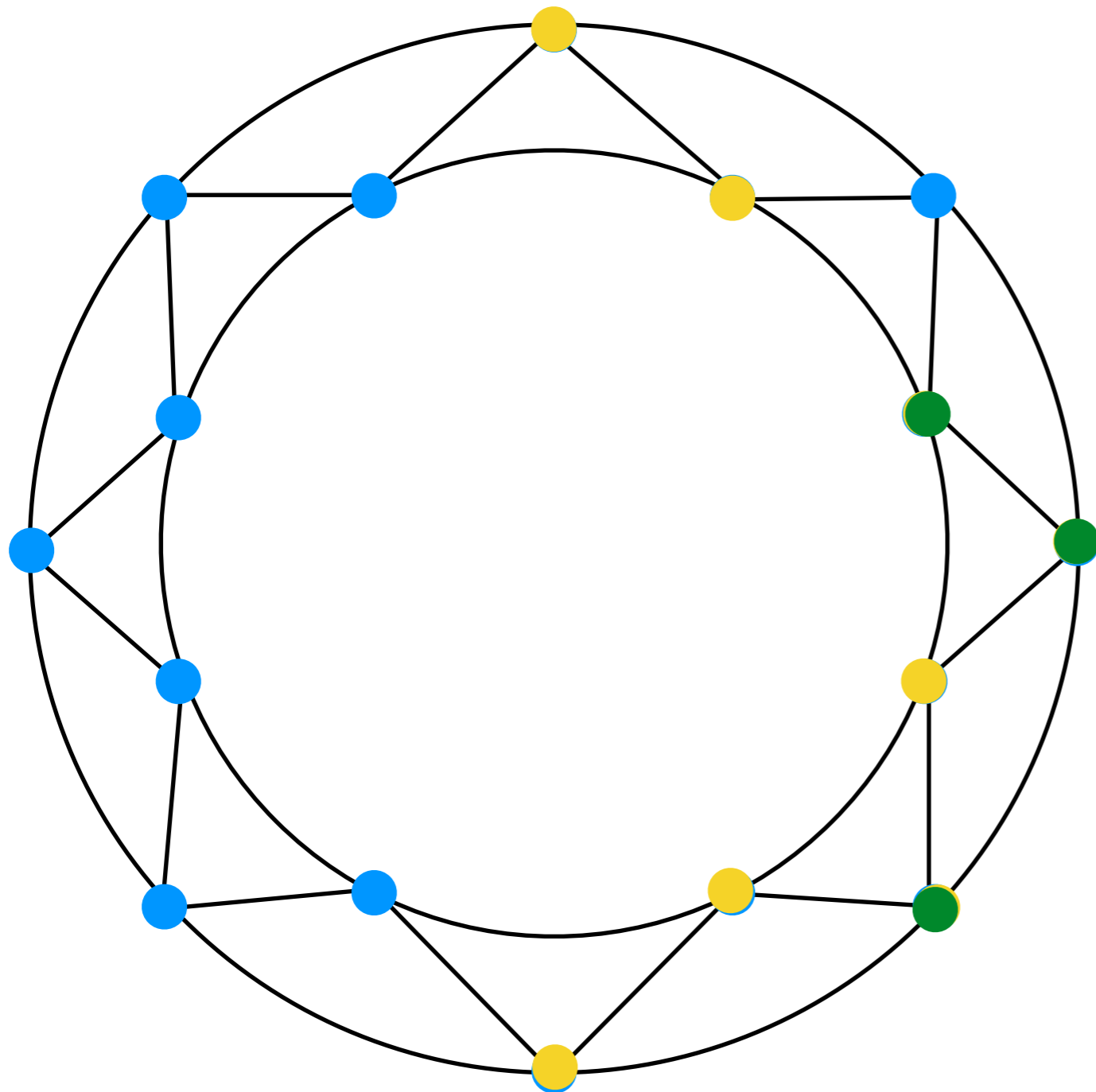
ref: Potterat et al (2002)



All Contacts

ref: orgnet.com

# SIR Model on a Network



Now what if the infection spreads on a network?

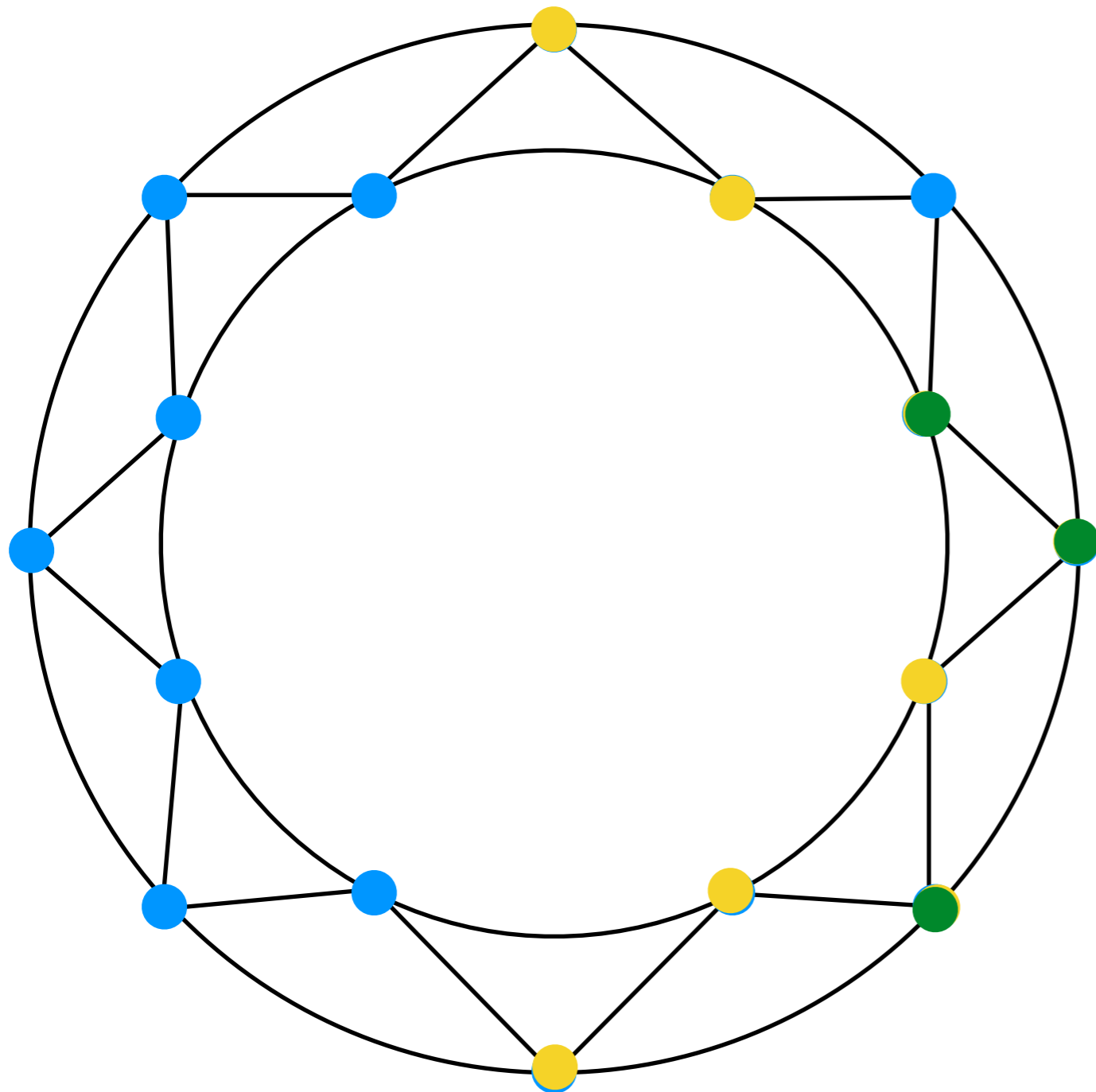
Still have S, I, R groups

Every time period, you are exposed to your direct neighbors

Prob(infected if friend is infected) =  $p$

Prob(recovery) =  $r$

# SIR Model on a Network

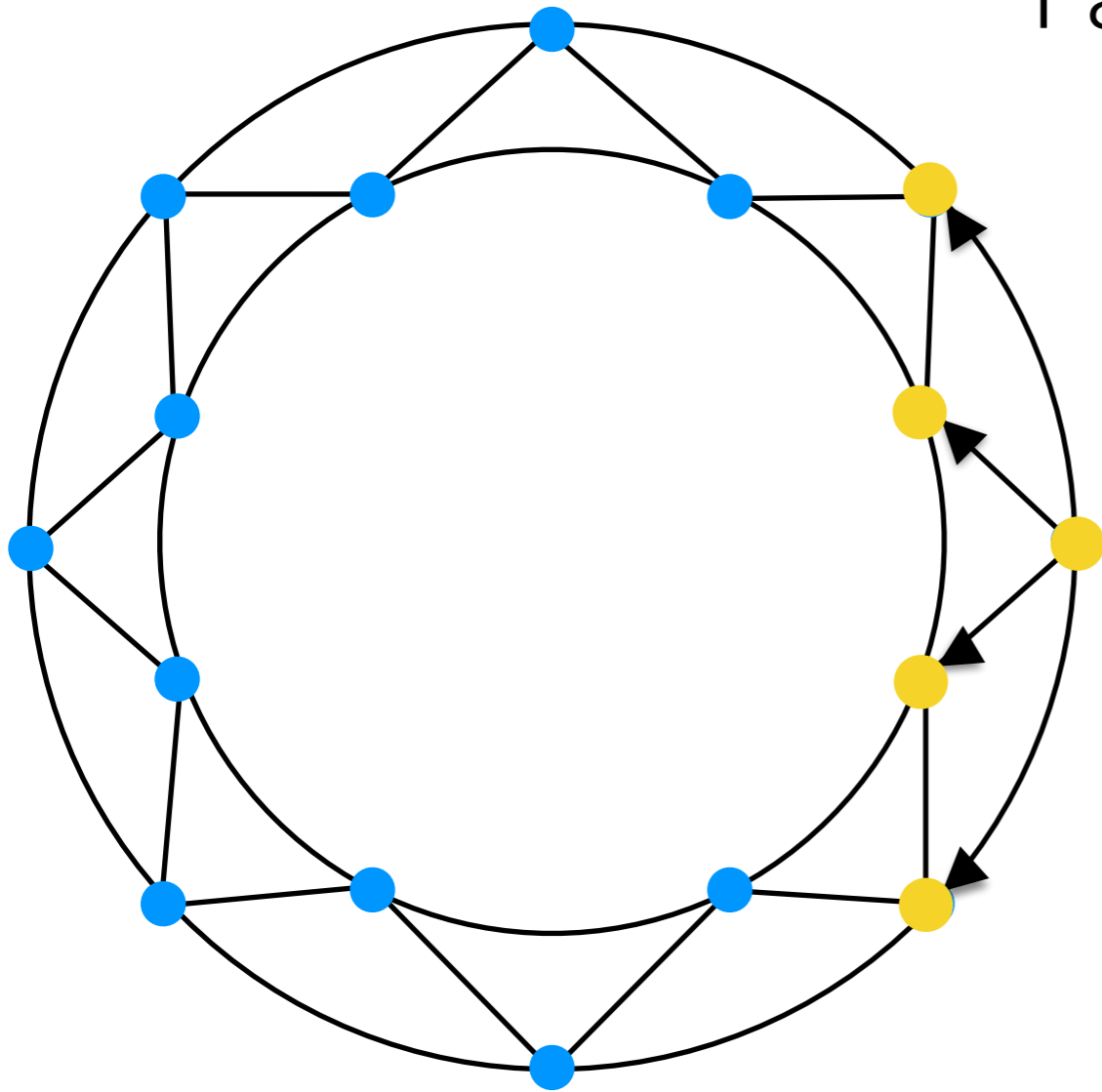


The probability of an epidemic still depends on the infection rate ( $p$ ) and the recovery rate ( $r$ ), but it also depends on network structure



# Network Structure and Diffusion

Factor 1: average degree

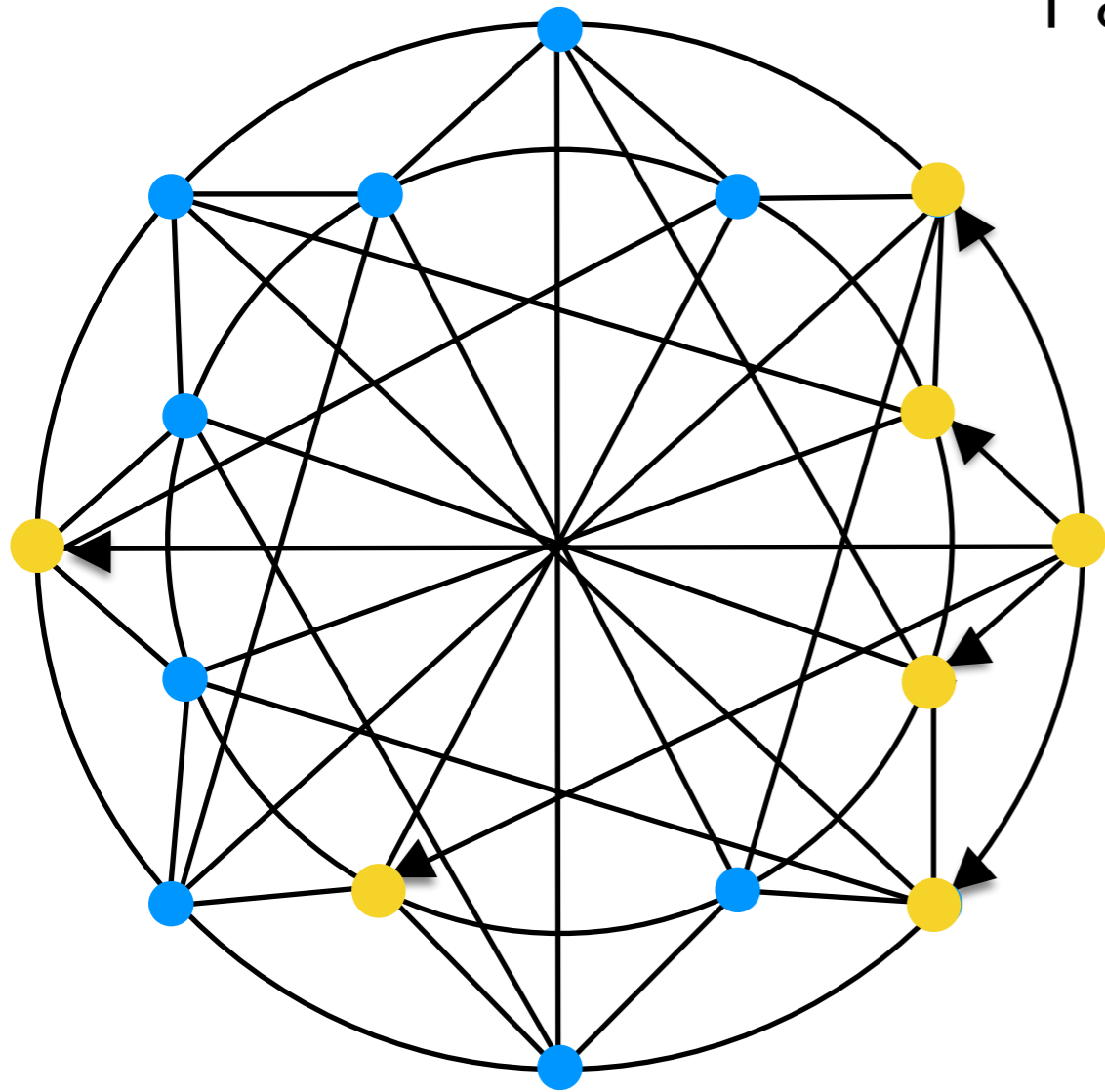


A node has the potential to infect everyone she is connected to

→ A higher average degree means more chances for an infected person to expose others

# Network Structure and Diffusion

Factor 1: average degree



A node has the potential to infect everyone she is connected to

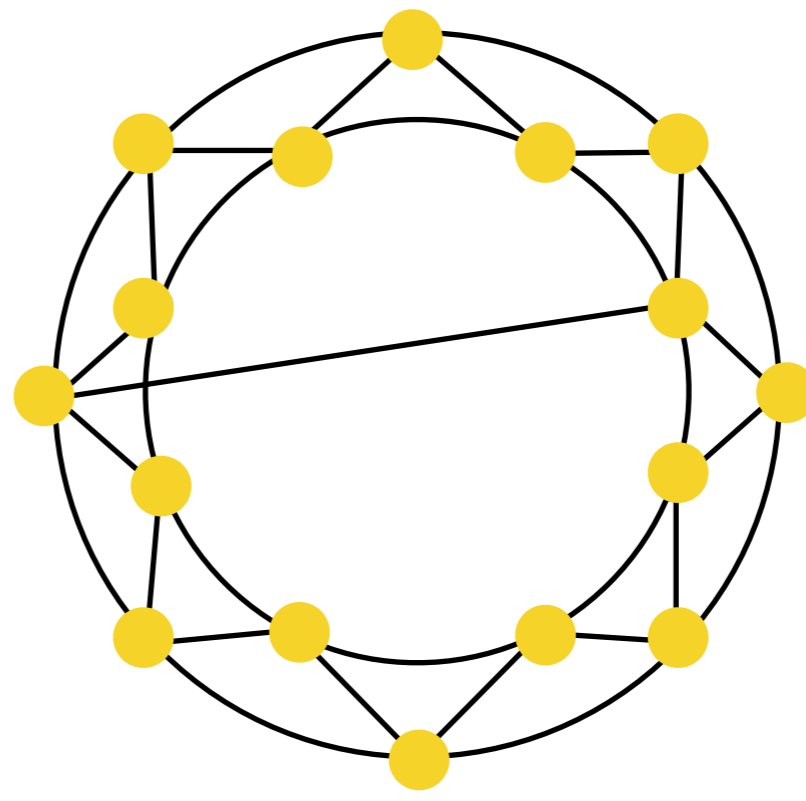
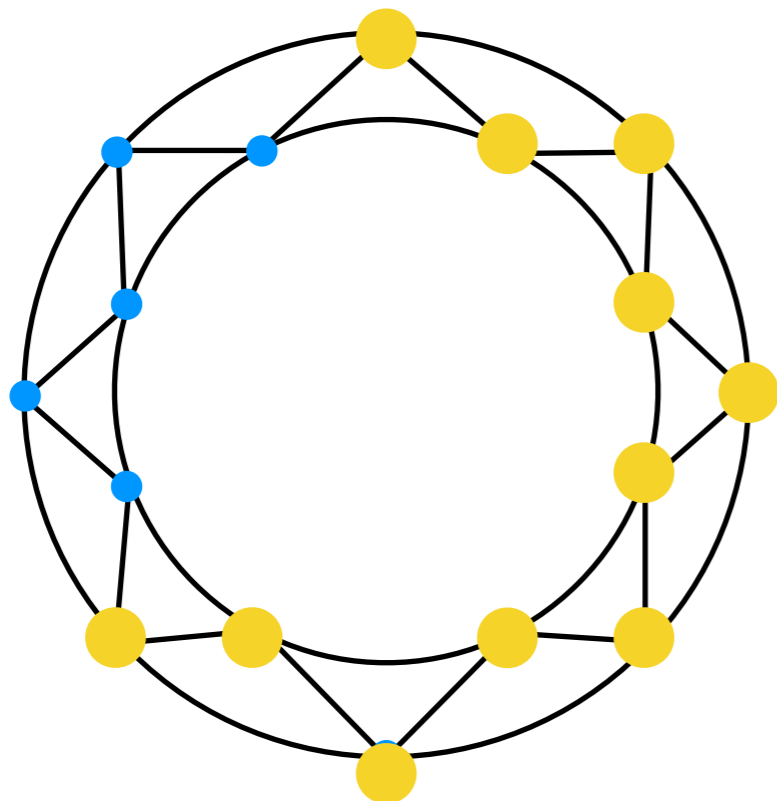
→ A higher average degree means more chances for an infected person to expose others

# Network Structure and Diffusion

## Factor 2: Average Distance

When people have contact with others in far-flung regions of the network, then disease can spread more widely.

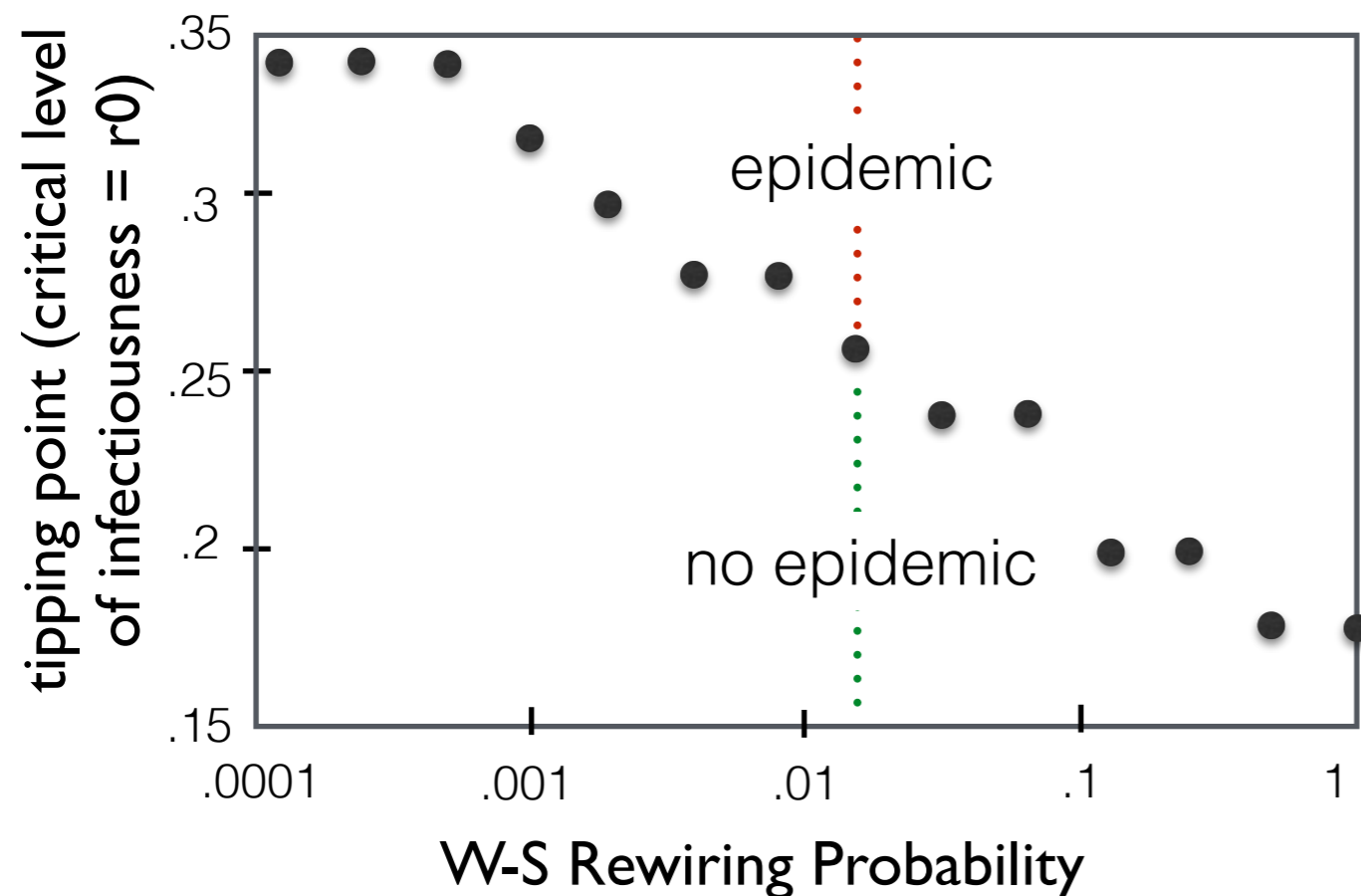
→ Epidemics are more likely in a small world



# Network Structure and Diffusion

Factor 2: Average Distance

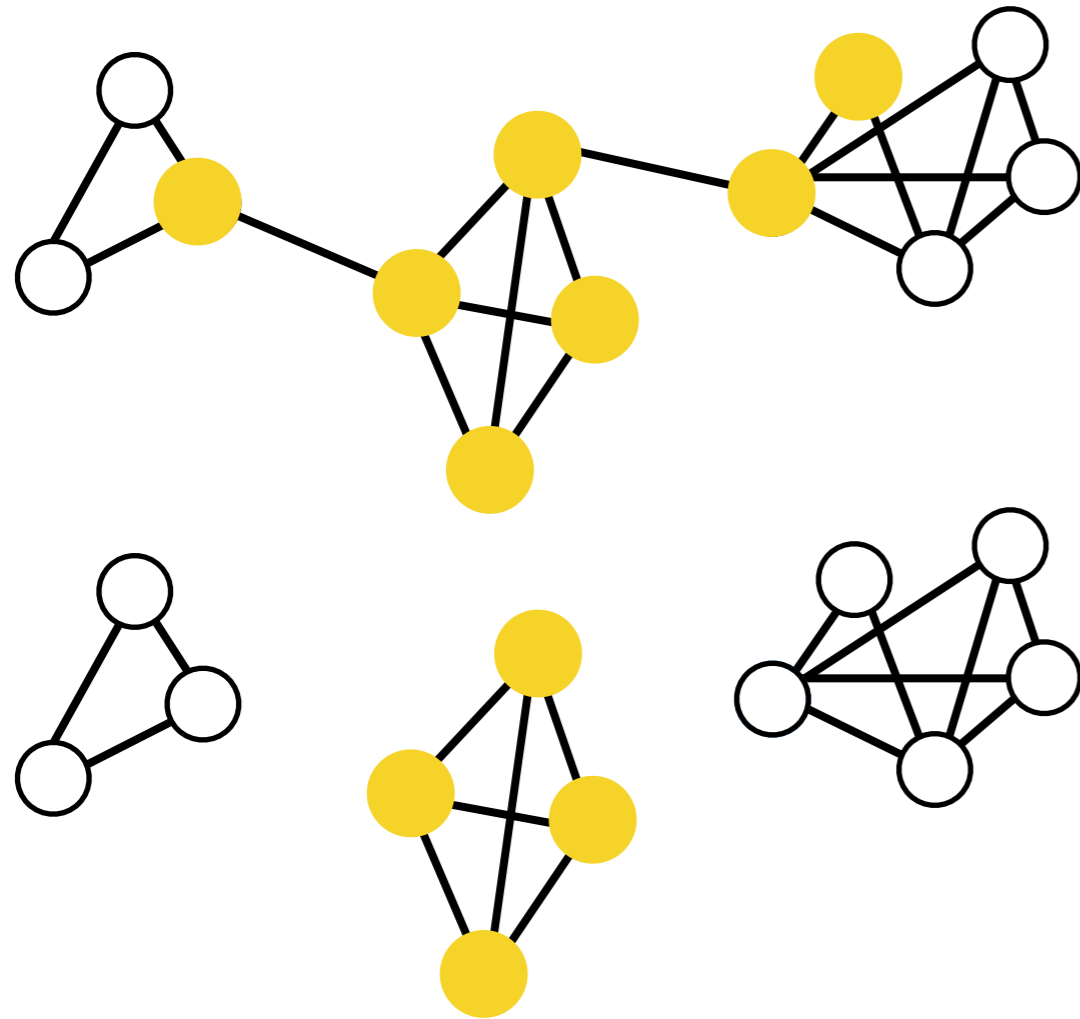
You can formalize this by looking at a Watts-Strogatz network



A few long-distance connections make it much easier for the disease to spread, which lowers the critical level of infectiousness ( $r_0$ ). This increases the likelihood of an epidemic.

# Network Structure and Diffusion

Factor 3: The size of the largest connected component



The disease can only spread to people who are connected to the original point of infection

→ disease will not spread as far when the network is broken into chunks

# What affects structure?

The structure of social networks that affect the spread of disease depends on characteristics of the disease, and characteristics of the society the disease lives within:

- Social and cultural norms

burial practices, greetings, condom use, sexual practices, norms of personal space, medical beliefs, family structure, visiting practices

- Environmental factors

sanitation systems, transportation systems, housing

- Health care policies

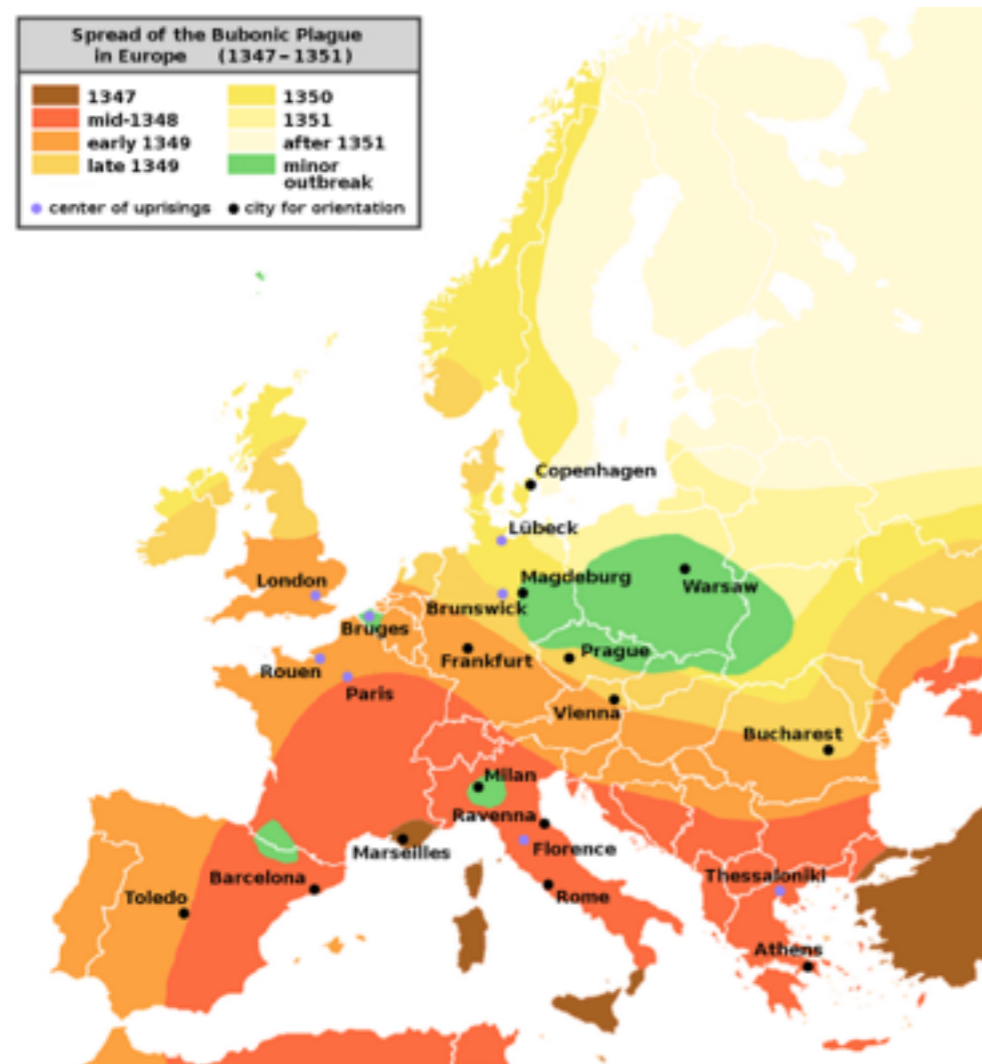
quarantine periods, immunization programs, awareness, public event cancelation, response time

- Transmission characteristics

type of contact required, duration of contact required, infectiousness, lifespan outside of host

# Network Structure and Diffusion

The effects of network structure can be seen in the differences between the spread of disease in the middle ages and the spread of disease today.



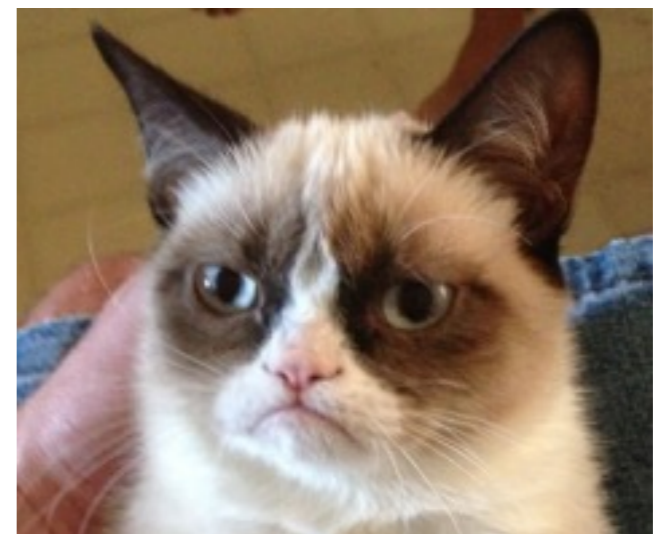
The spread of the plague was certainly terrifying in its time, but it's nothing compared to the pandemics of today's more connected world...

# Network Structure and Diffusion

Although we haven't yet talked about the diffusion of *ideas*, you can also see the effects of network structure in the spread of viral content

Fads have existed forever. But they spread more quickly and more widely than they used to.

→ “Epidemics” of viral content have become “pandemics”





# Controlling Diffusion

The relationship between network structure and diffusion suggests ways that we could prevent epidemics

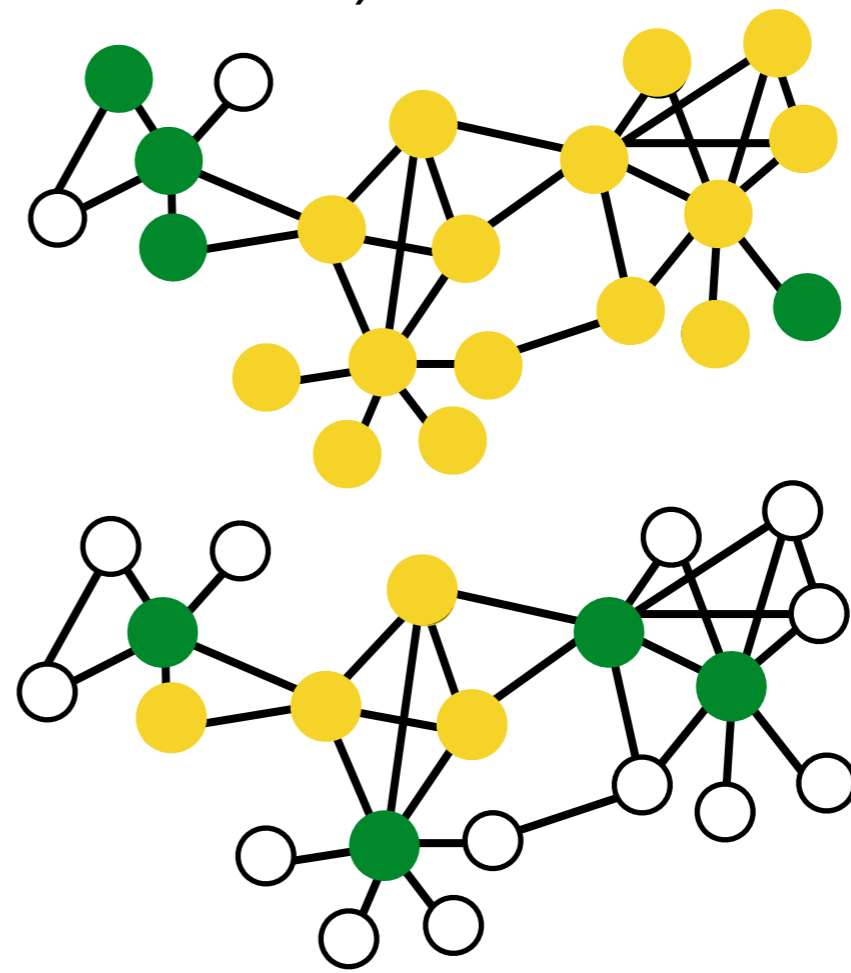
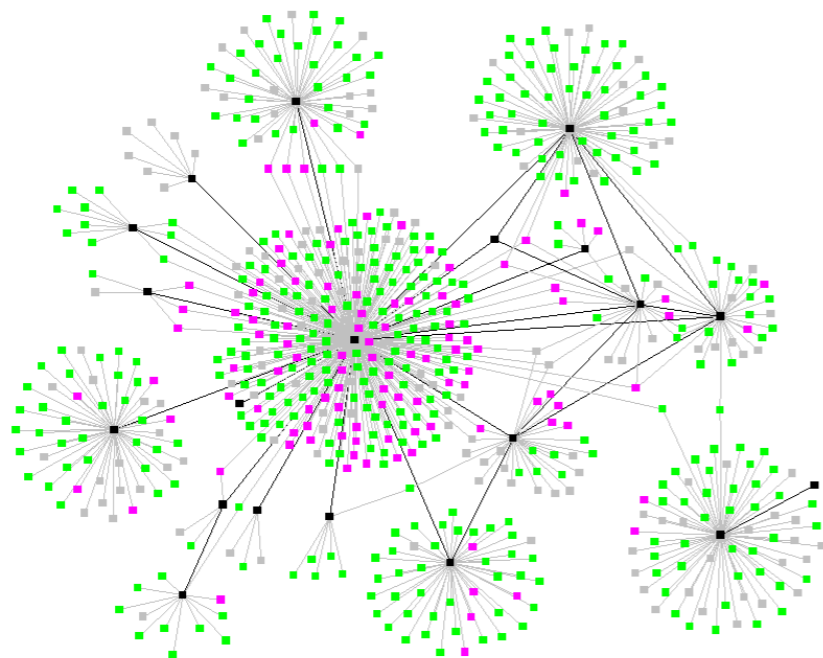
- change who interacts with who
- change who is susceptible
- change who is infected

# Controlling Diffusion: Disease

Vaccinate a small number of people

→ Vaccinating the right people can prevent the spread of the disease with very few resources

→ Good people to vaccinate are hubs (health care workers) and bridges (bus drivers)

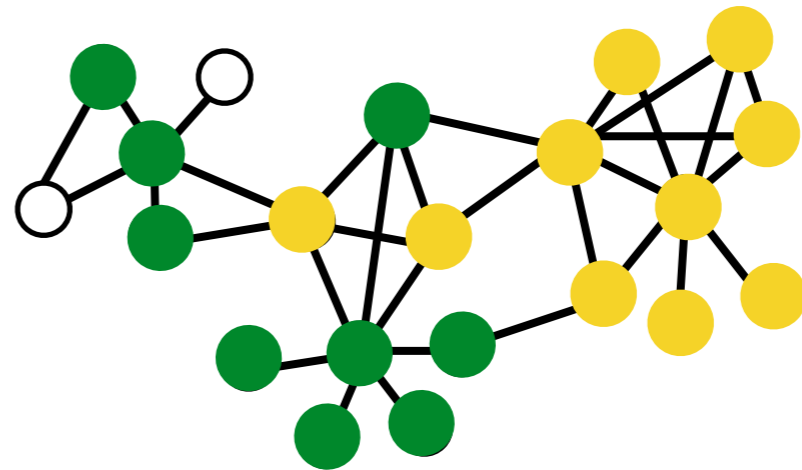
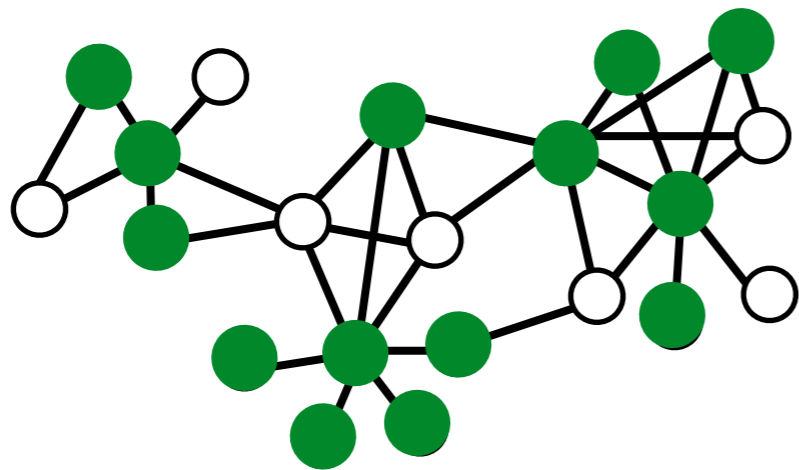


# Controlling Diffusion: Disease

Vaccinate a large number of people

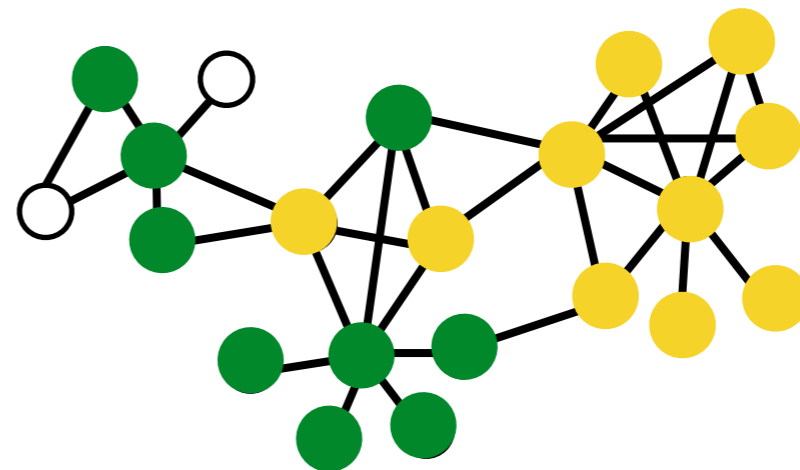
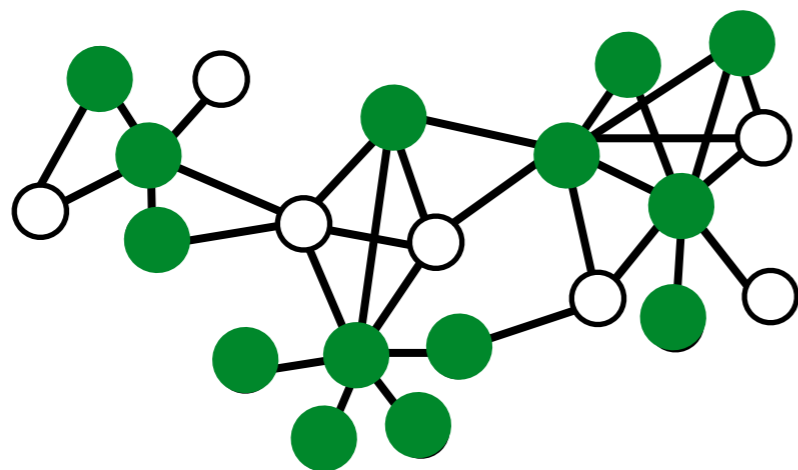
→ Vaccinating enough people effectively reduces the size of the largest component to 0

→ The fact that the network is broken up protects people who are unable to be vaccinated. This is called “herd immunity”.



# Controlling Diffusion: Disease

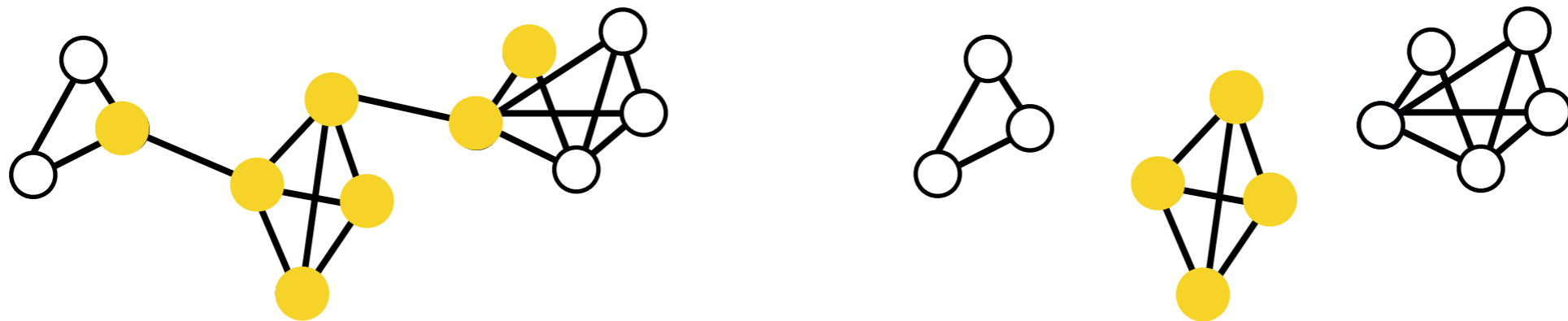
- If the population is vaccinated for long enough, the disease won't have enough hosts and will die out (eg: small pox)
- Subpopulations that fail to vaccinate may lose herd immunity and have outbreaks, which may infect people in other communities who can't be vaccinated



# Controlling Diffusion: Disease

## Quarantine

- Quarantines break up the network into much smaller chunks (eg: households)
- Limiting large gatherings (eg: baseball games) and travel restrictions can achieve the same thing.



# Summary

Takeaways from today:

- Diffusion is how things like disease and information spread through a population
- A lot of our knowledge of diffusion comes from epidemiology—the SI model and the SIR model
- Diffusion on a network gives us some ideas of how to start to control the process

Next time:

- Information: fads, virality, network effects, strategy, and cascades