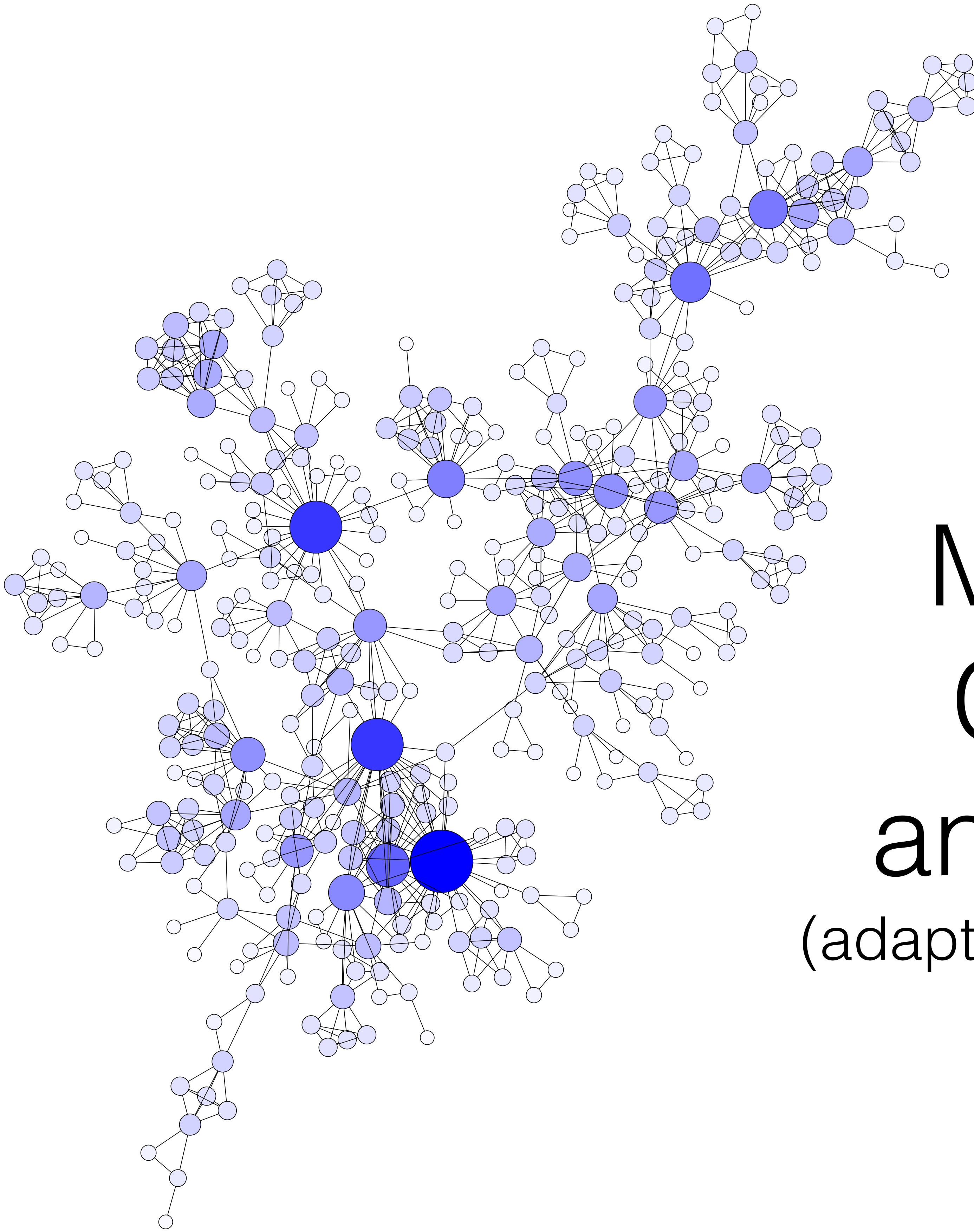
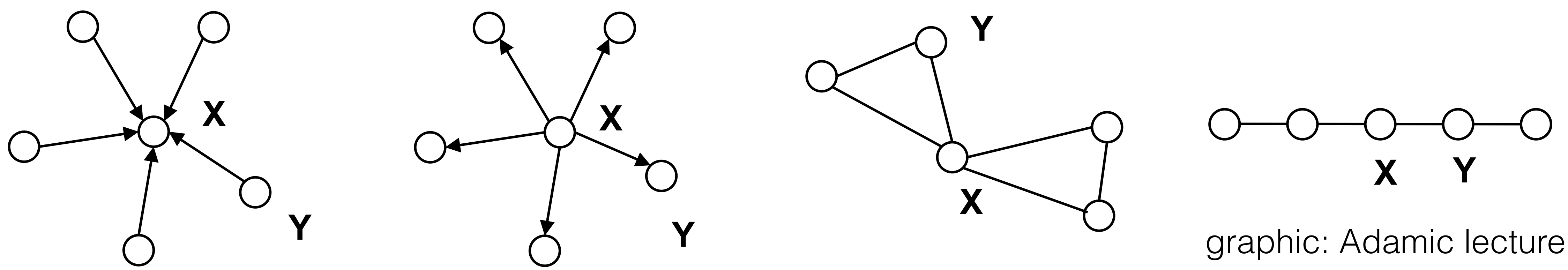


Network Measures: Centrality and Prestige

(adapted from Lada Adamic)

Centrality and Prestige

Some nodes are more important than others



graphic: Adamic lecture

But what it means to be "important" depends on the context: exchange, spread of information, brokerage opportunities, etc.

Centrality measures give us a way to quantify the different ways that a node can be important

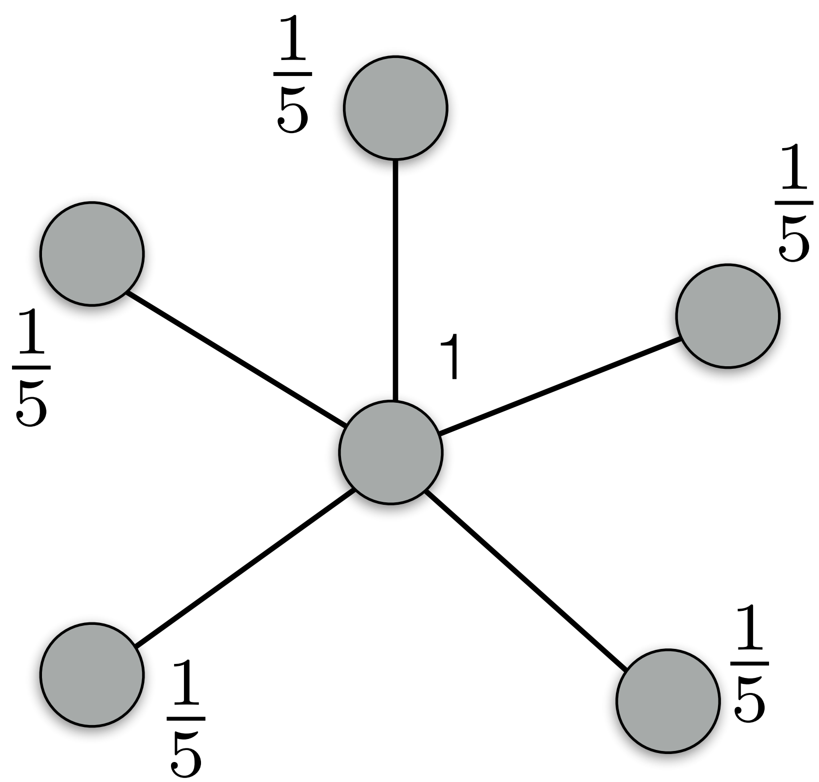
Centrality and Prestige

Today:

- Tour through a variety of centrality measures:
    - Degree
    - Betweenness
    - Closeness
    - Eigenvector
- Look at how centrality is distributed: centralization
- Centrality on a directed network: prestige

Degree Centrality

- First notion: the person with the most connections is most important

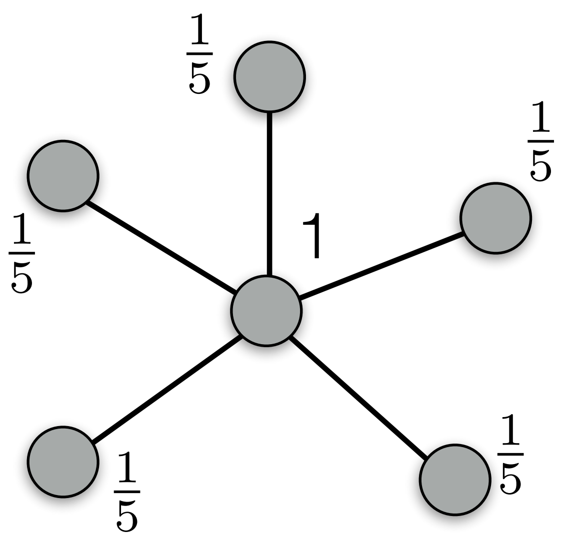


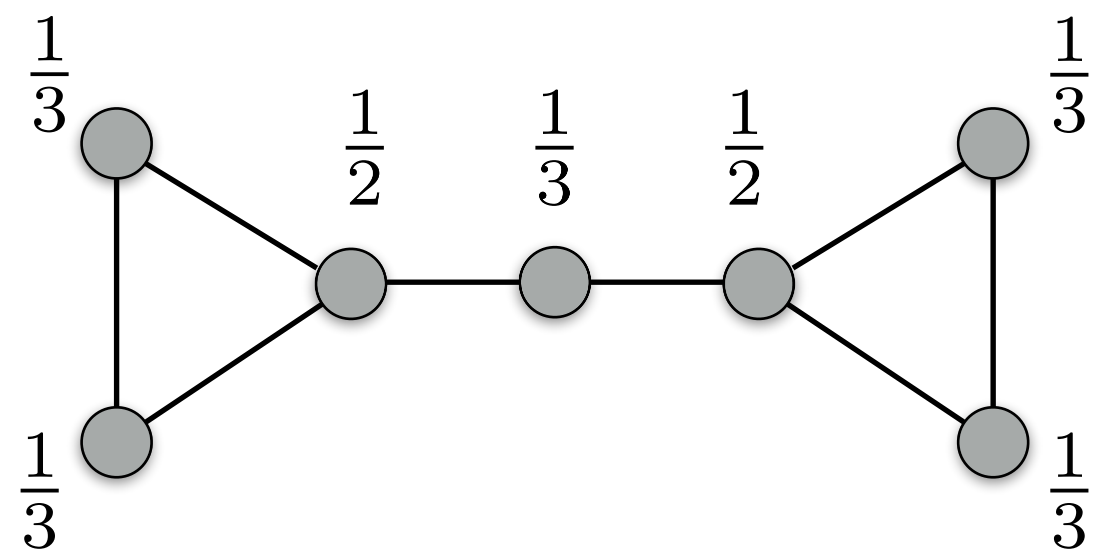
$$C_d(v_i) = \frac{1}{N-1} d_i$$

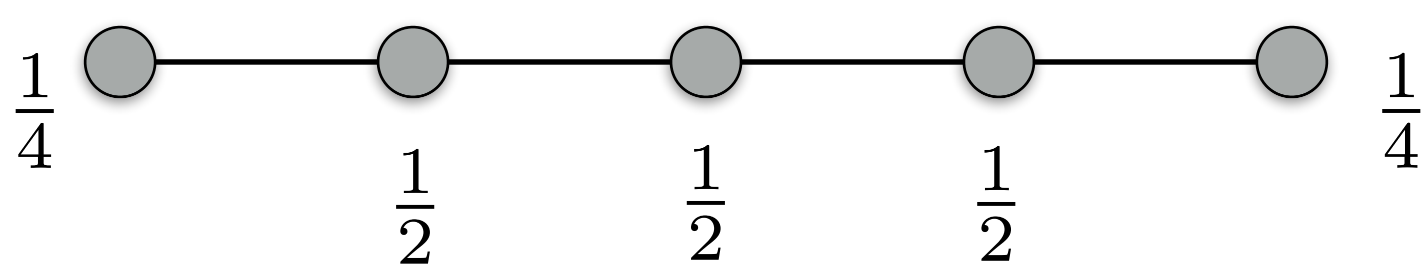
Normalize by the maximum possible number of connections you could have: (N-1)

Degree Centrality

$$C_d(v_i) = \frac{1}{N-1} d_i$$

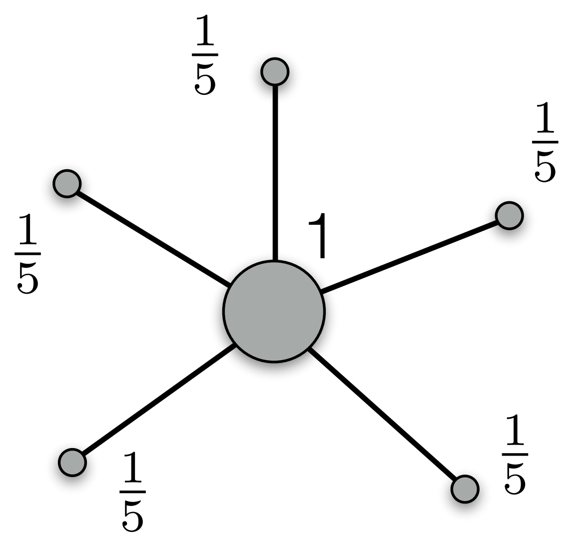


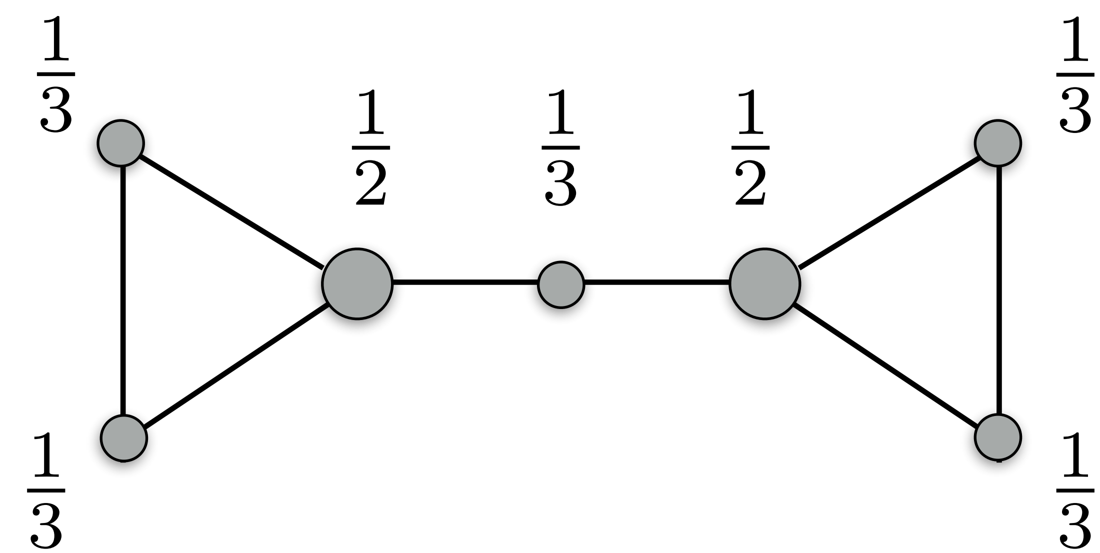


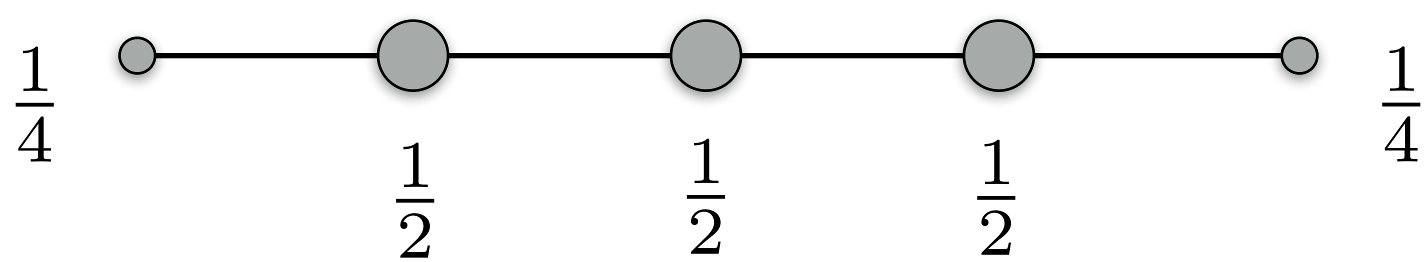


Degree Centrality

$$C_d(v_i) = \frac{1}{N-1} d_i$$

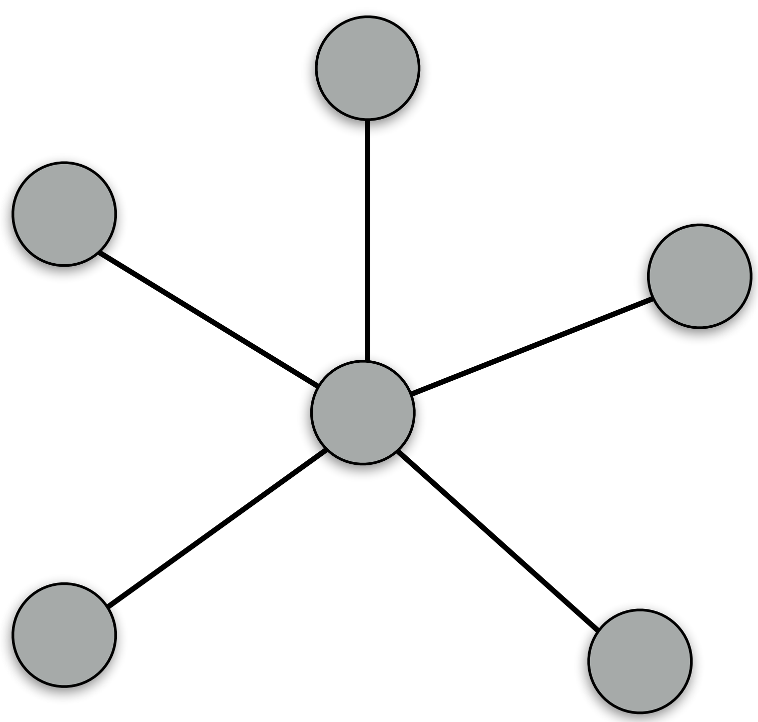






Degree Centrality

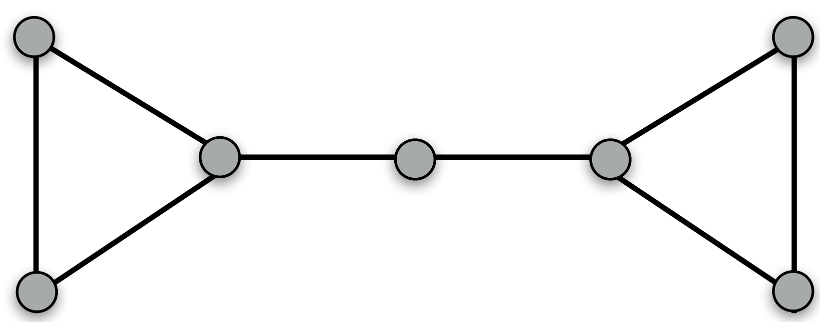
- Degree centrality makes sense when sheer *number* of contacts is important:

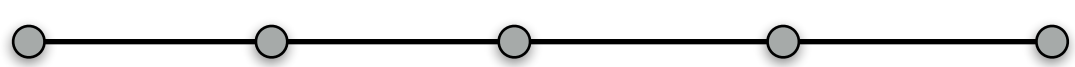


- Number of supporters
- Number of confidants
- Audience size
- Number of trading partners
- Number of direct reports

Degree Centrality

- Clearly, there are some contexts where degree isn’t exactly what we mean by “centrality”





- Suppose we are interested in who gets access to information?
- Or who can broker between different groups?

Closeness Centrality

- Second notion: the person in the middle of the action is most central

$$C_C(v_i) = \frac{(N-1)}{\sum_{v_j \in G} d(v_i, v_j)}$$

Normalization (min possible distance to the N-1 other nodes)

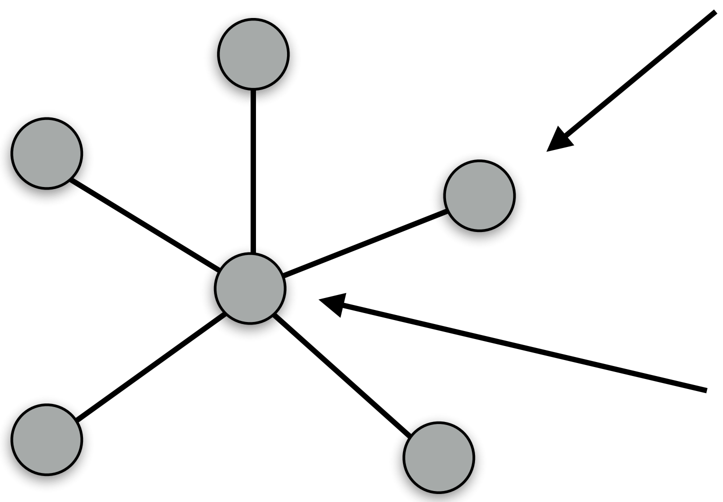
$d\left(v_i, v_j\right) =$ distance btwn i *and* j

Total distance btwn i and the other nodes

- Person with the highest closeness centrality has the shortest average distance to other nodes

Closeness Centrality

$$C_C(v_i) = \frac{(N-1)}{\sum_{v_j \in G} d(v_i, v_j)}$$

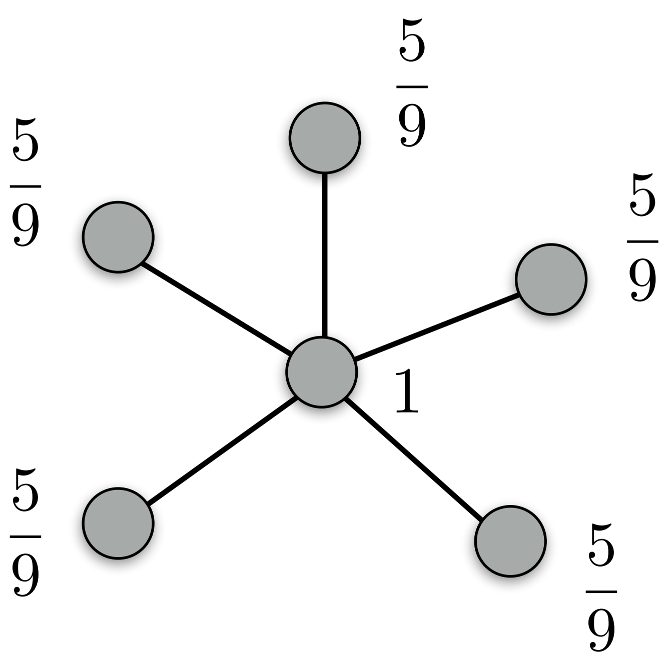


$$C_C(v_i) = \frac{5}{1 + 4(2)} = \frac{5}{9}$$

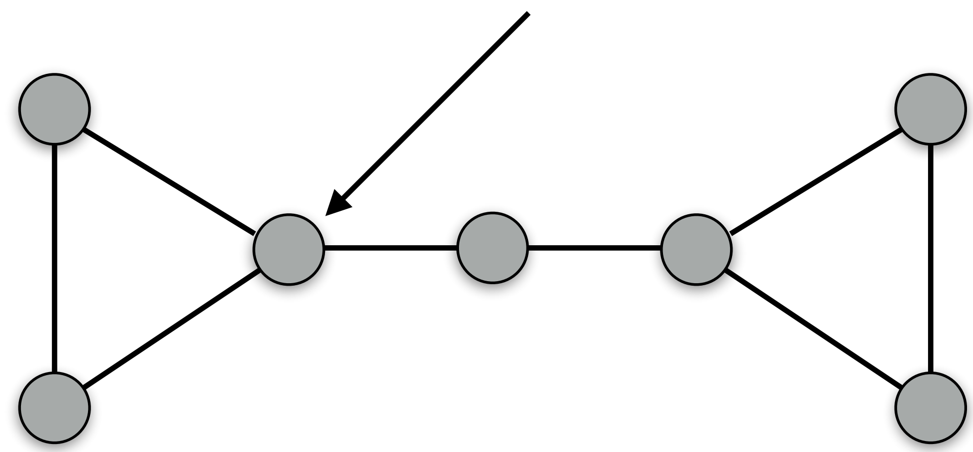
$$C_C(v_i) = \frac{N-1}{\sum_{v_j \in G} 1} = \frac{5}{5}$$

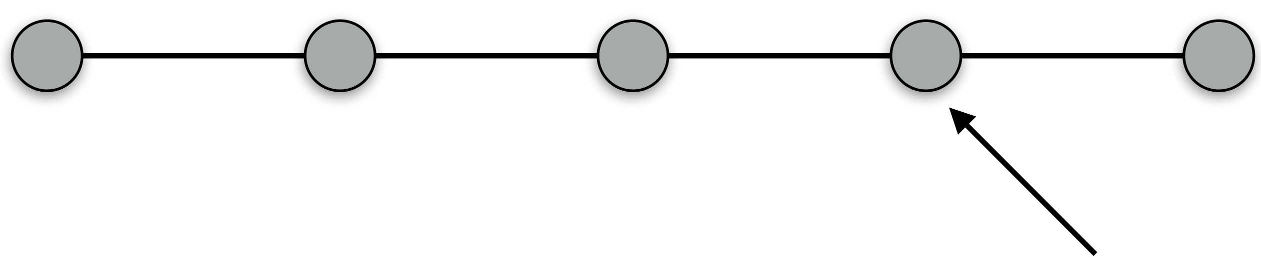
Closeness Centrality

$$C_C(v_i) = \frac{(N-1)}{\sum_{v_j \in G} d(v_i, v_j)}$$



$$C_C(v_i) = \frac{6}{3(1) + 1(2) + 2(3)} = \frac{6}{11}$$

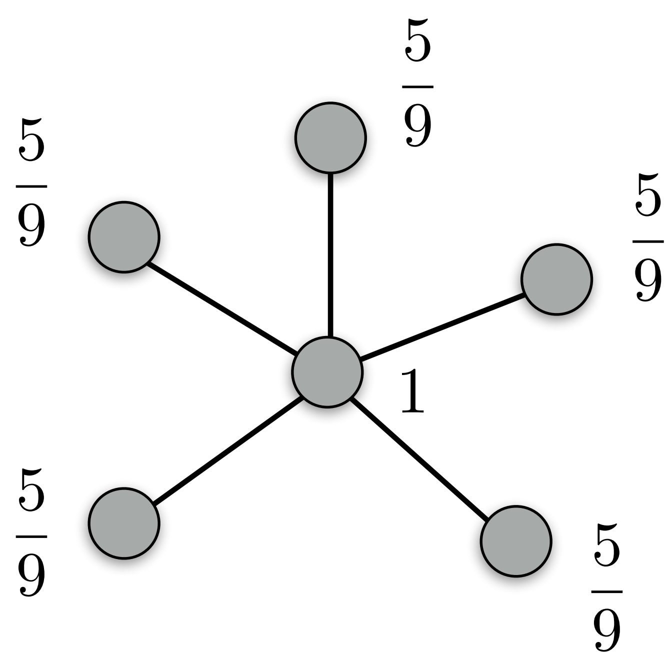


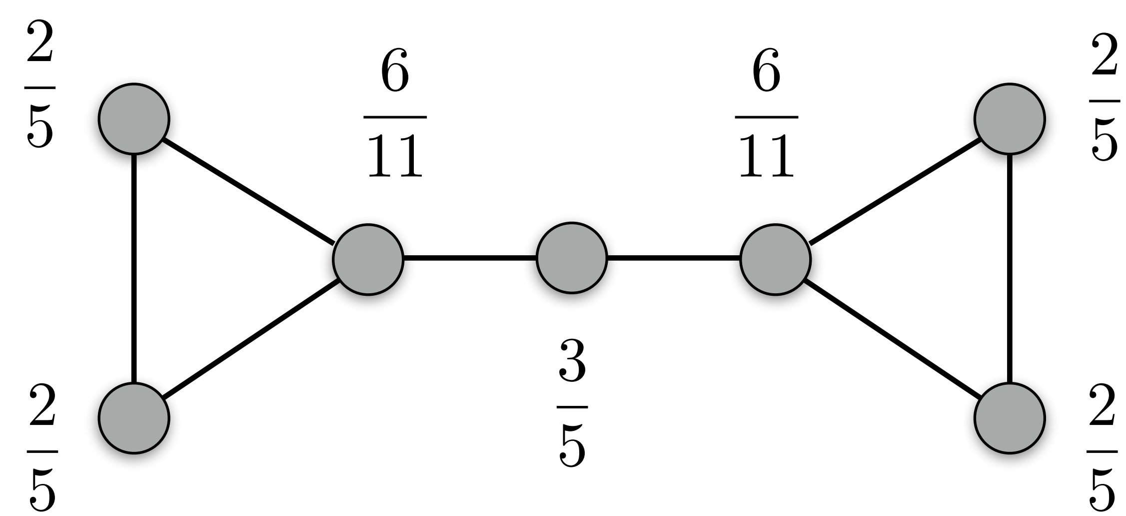


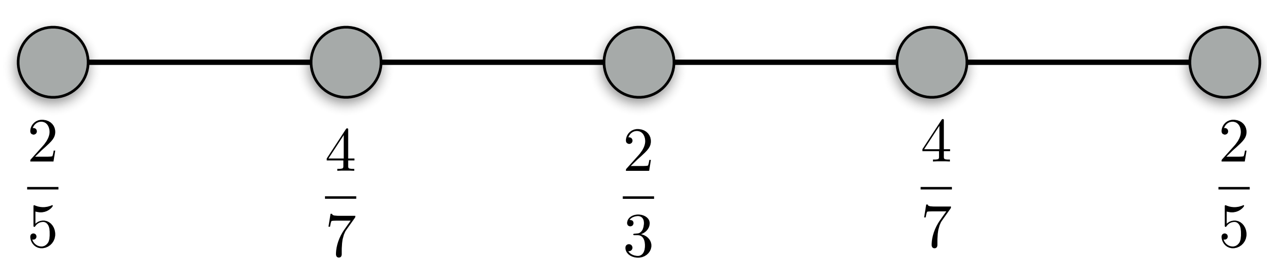
$$C_C(v_i) = \frac{4}{2(1) + 1(2) + 1(3)} = \frac{4}{7}$$

Closeness Centrality

$$C_C(v_i) = \frac{(N-1)}{\sum_{v_j \in G} d(v_i, v_j)}$$

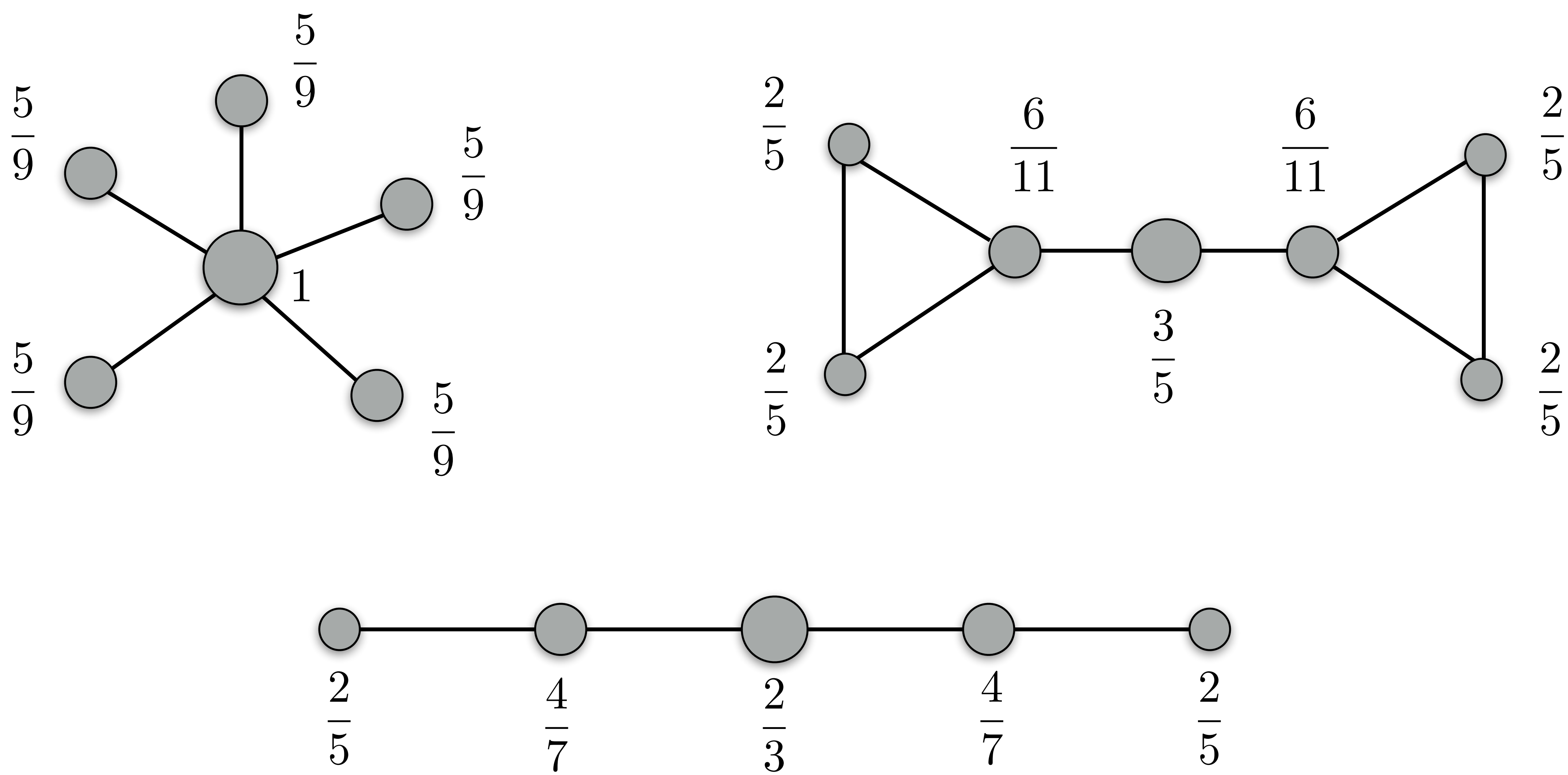






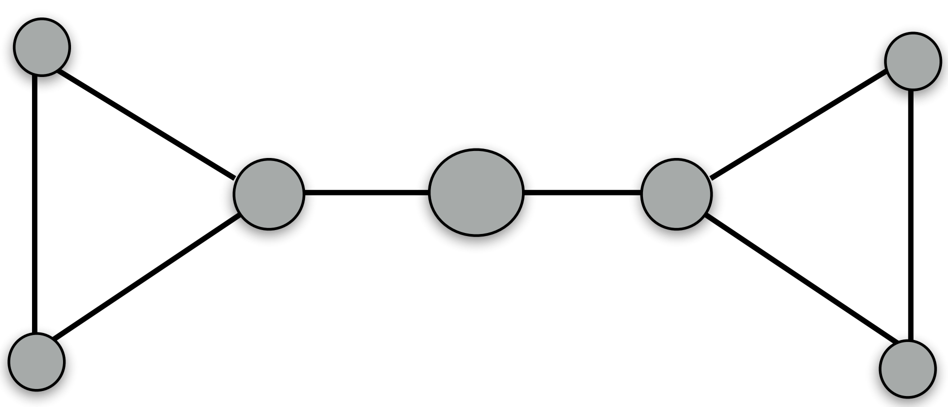
Closeness Centrality

$$C_C(v_i) = \frac{(N-1)}{\sum_{v_j \in G} d(v_i, v_j)}$$



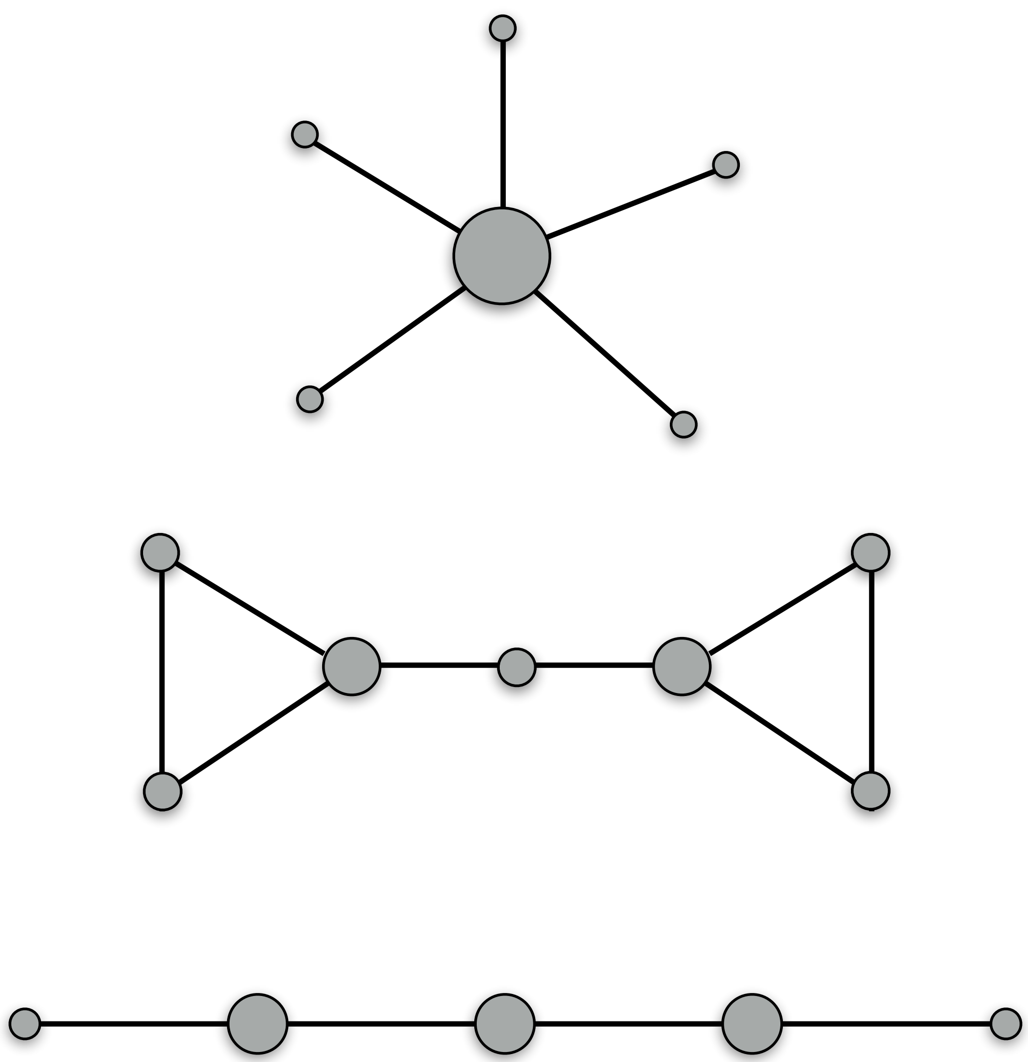
Closeness Centrality

- Closeness centrality makes sense whenever *direct* access is important

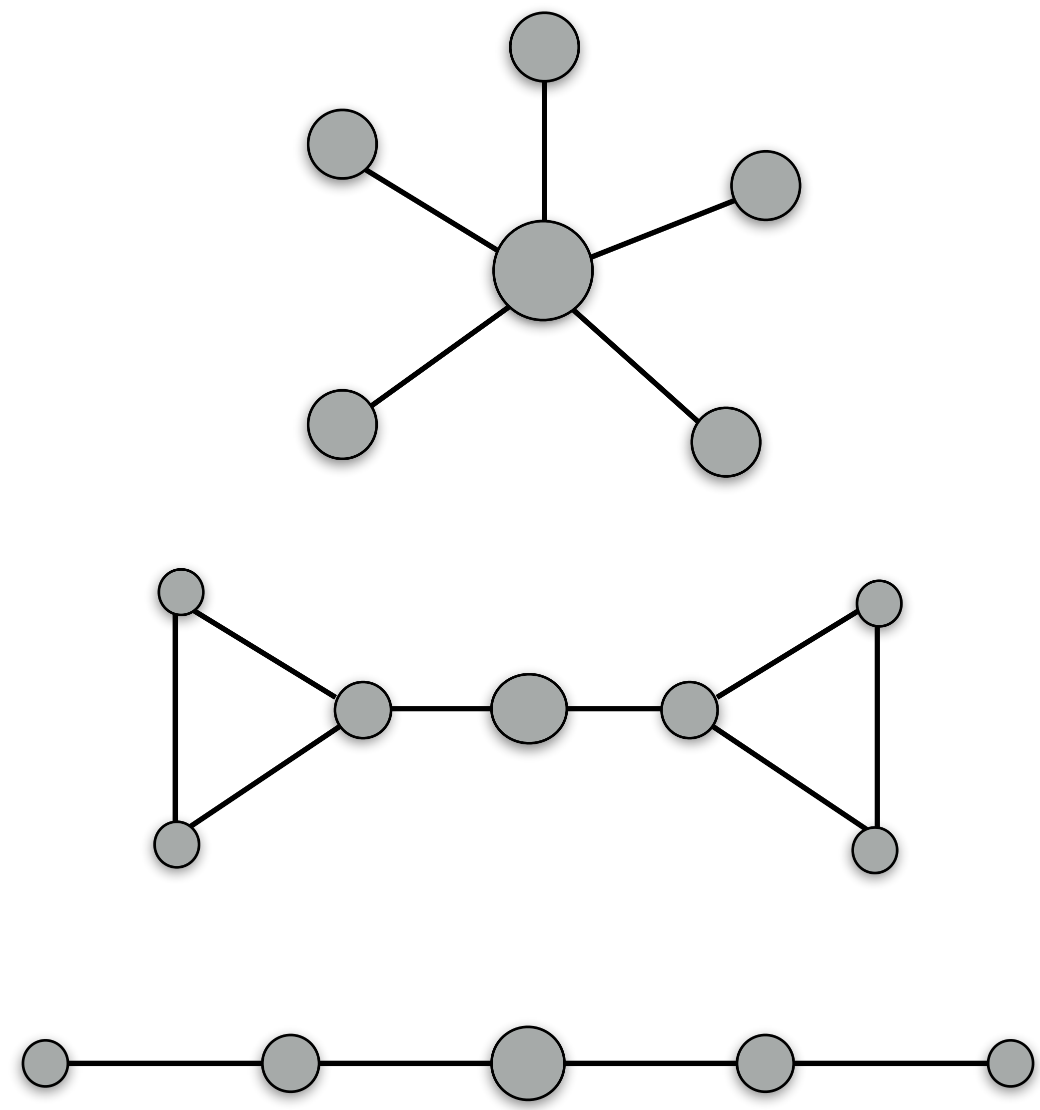


- Access to information
- Opinion formation
- Spread of disease
- Adoption of new technology

Degree vs Closeness



Degree



Closeness

Betweenness Centrality

Third notion: the most important people are those you have to go through to get to others

So what fraction of the geodesics go through the node?

$g_{jk}(v_i)$ = number of geodesics between j and k that go through i

$$C_B(v_i) = \frac{\sum_{j<k} \frac{g_{jk}(v_i)}{g_{jk}}}{(N-1)(N-2)/2}$$

g_{jk} = total number of geodesics between j and k

Betweenness Centrality

Third notion: the most important people are those you have to go through to get to others

So what fraction of the geodesics go through the node?

fraction of geodesics between j and k that go through i

$$C_B(v_i) = \frac{\sum_{j<k} \frac{g_{jk}(v_i)}{g_{jk}}}{(N-1)(N-2)/2}$$

Betweenness Centrality

Third notion: the most important people are those you have to go through to get to others

So what fraction of the geodesics go through the node?

$$C_B(v_i) = \frac{\sum_{j<k} \frac{g_{jk}(v_i)}{g_{jk}}}{(N-1)(N-2)/2}$$

total fraction of geodesics that go through node i

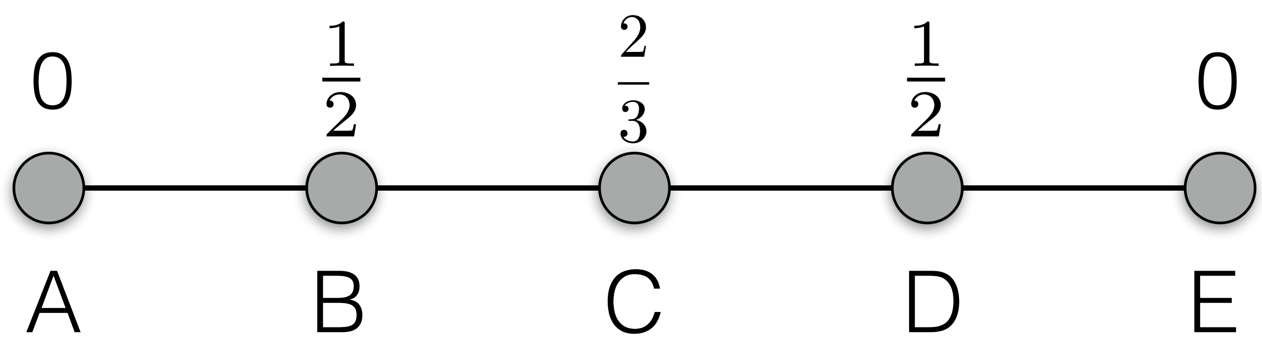
fraction of geodesics between j and k that go through i

Normalization: $\binom{N-1}{2}$

If all geodesics between all pairs of nodes go through node i

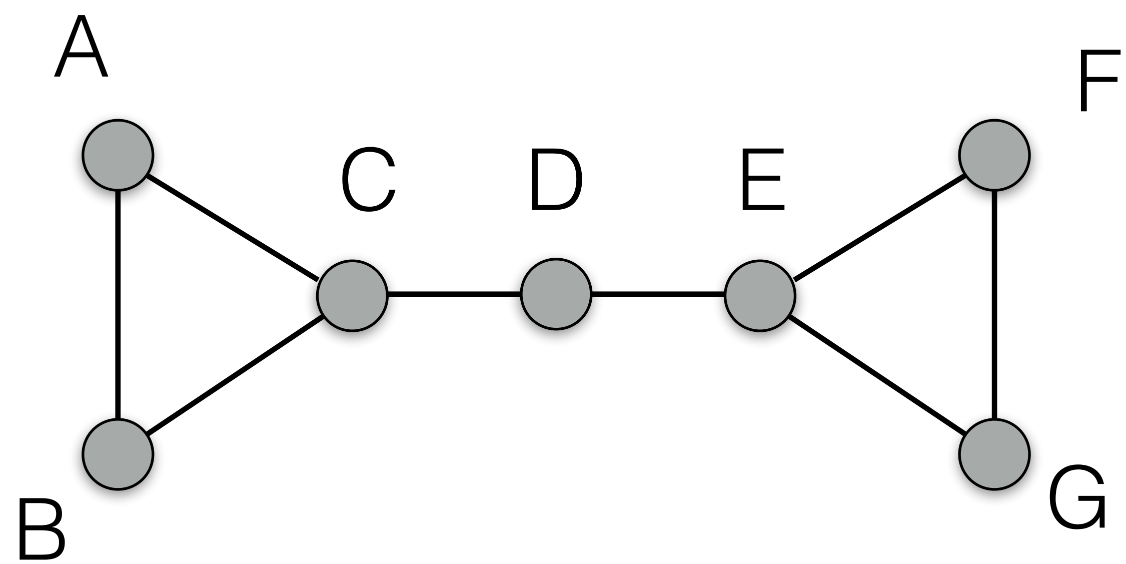
Betweenness Centrality

$$C_B(v_i) = \frac{\sum_{j<k} \frac{g_{jk}(v_i)}{g_{jk}}}{(N-1)(N-2)/2}$$



- A and E are not on any shortest paths
- B and D are both on 3 shortest paths
- C is on 4 shortest paths

Betweenness Centrality



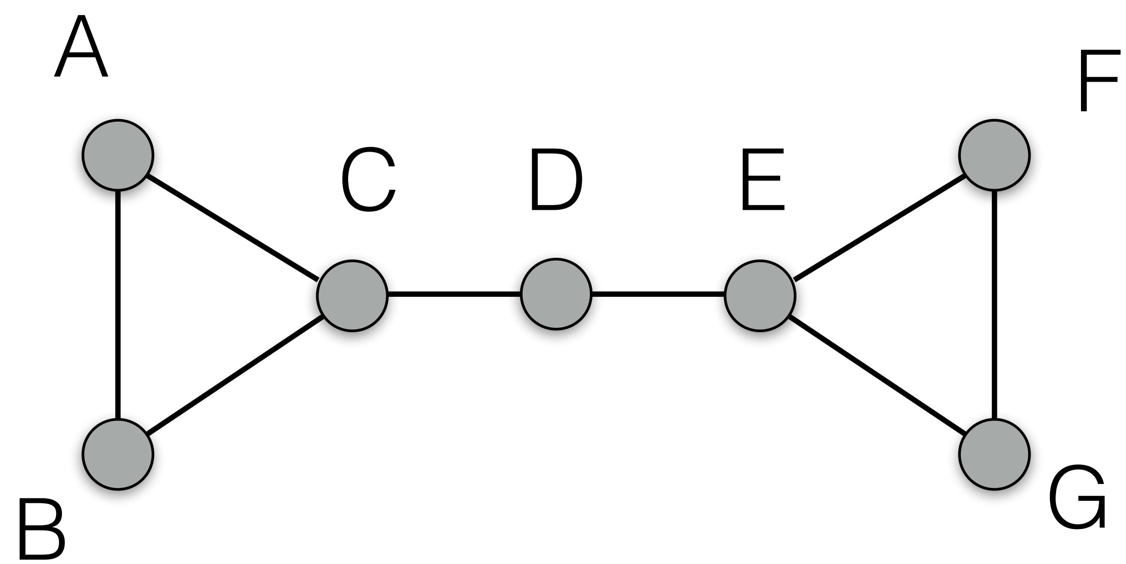
$$C_B(v_i) = \frac{\sum_{j<k} \frac{g_{jk}(v_i)}{g_{jk}}}{(N-1)(N-2)/2}$$

A:

BC	$\frac{0}{1}$	CD	$\frac{0}{1}$	DE	$\frac{0}{1}$	EF	$\frac{0}{1}$	FG	$\frac{0}{1}$
BD	$\frac{0}{1}$	CE	$\frac{0}{1}$	DF	$\frac{0}{1}$	EG	$\frac{0}{1}$		
BE	$\frac{0}{1}$	CF	$\frac{0}{1}$	DG	$\frac{0}{1}$				
BF	$\frac{0}{1}$	CG	$\frac{0}{1}$						
BG	$\frac{0}{1}$								

$$\rightarrow C_B(A) = \frac{15 * 0}{15}$$

Betweenness Centrality



$$C_B(v_i) = \frac{\sum_{j<k} \frac{g_{jk}(v_i)}{g_{jk}}}{(N-1)(N-2)/2}$$

D:

AB $\frac{0}{1}$

AC $\frac{0}{1}$

AE $\frac{1}{1}$

AF $\frac{1}{1}$

AG $\frac{1}{1}$

BC $\frac{0}{1}$

BE $\frac{1}{1}$

BF $\frac{1}{1}$

BG $\frac{1}{1}$

CE $\frac{1}{1}$

CF $\frac{1}{1}$

CG $\frac{1}{1}$

EF $\frac{0}{1}$

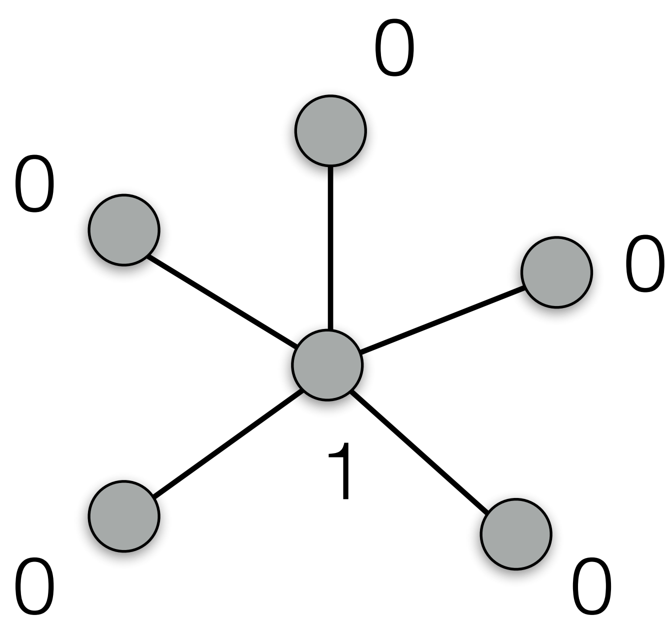
EG $\frac{0}{1}$

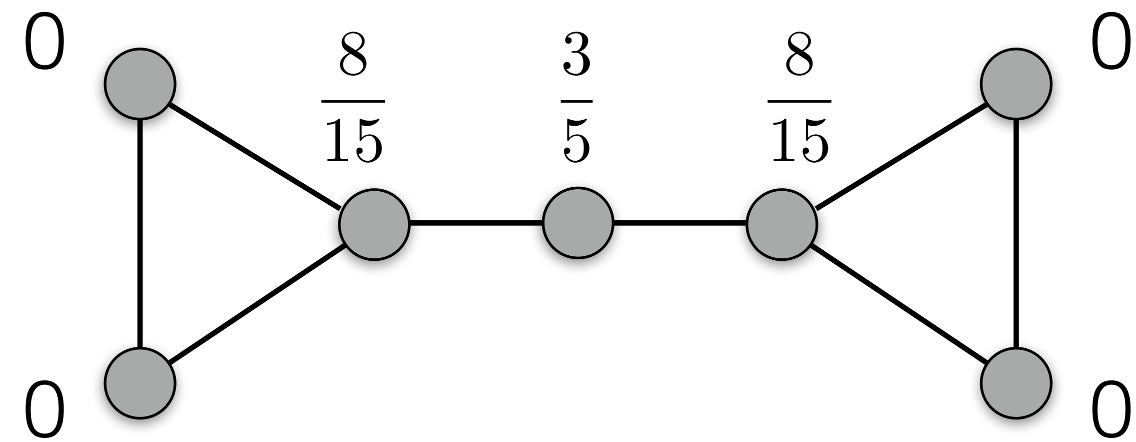
FG $\frac{0}{1}$

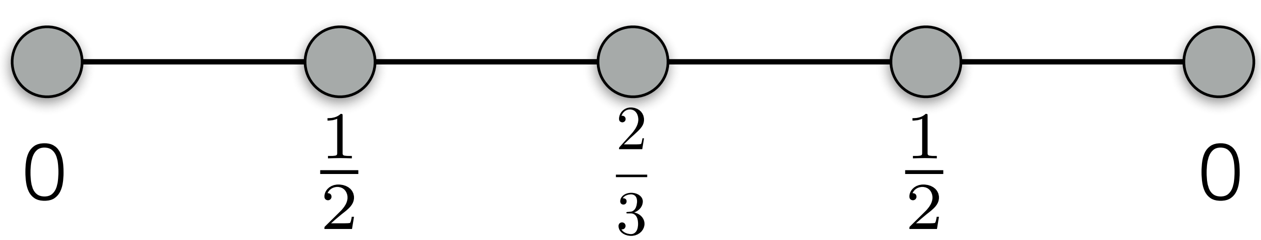
$$\rightarrow C_B(D) = \frac{6*0 + 9*1}{15} = \frac{3}{5}$$

Betweenness Centrality

$$C_B(v_i) = \frac{\sum_{j<k} \frac{g_{jk}(v_i)}{g_{jk}}}{(N-1)(N-2)/2}$$

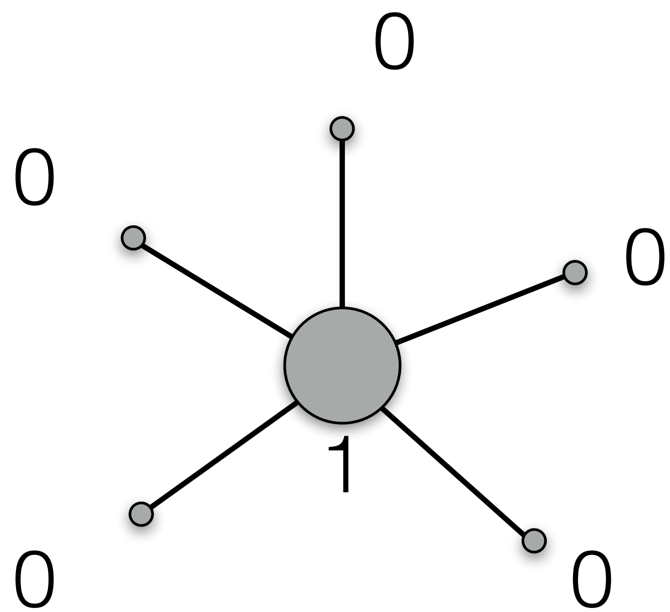


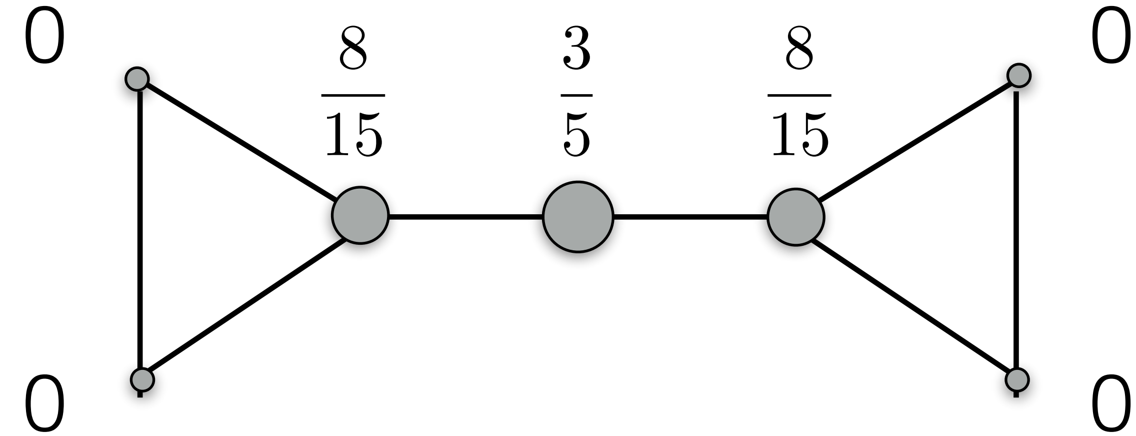


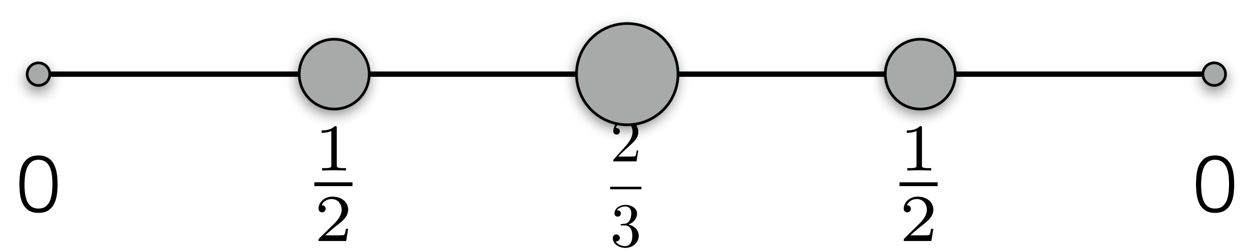


Betweenness Centrality

$$C_B(v_i) = \frac{\sum_{j<k} \frac{g_{jk}(v_i)}{g_{jk}}}{(N-1)(N-2)/2}$$

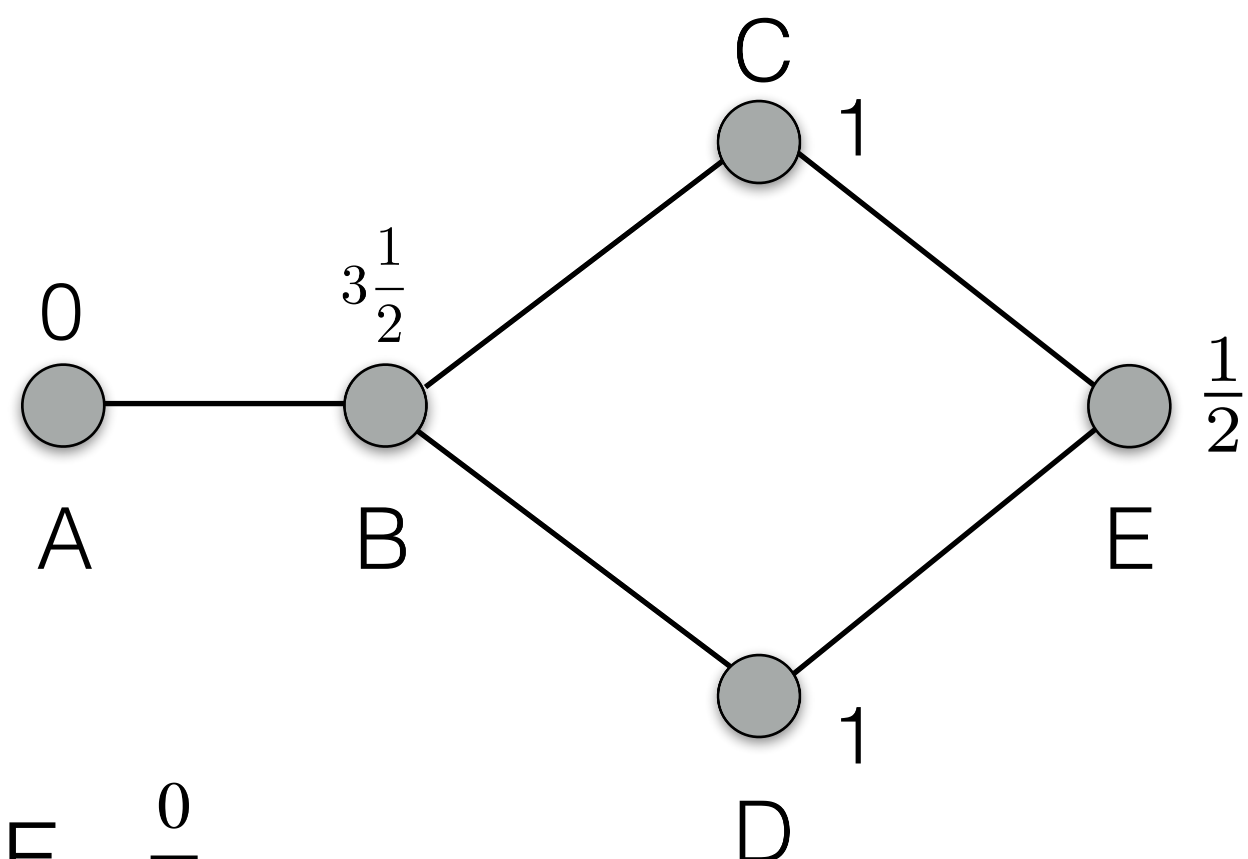






Betweenness Centrality

$$C_B(v_i) = \frac{\sum_{j<k} \frac{g_{jk}(v_i)}{g_{jk}}}{(N-1)(N-2)/2}$$



AC $\frac{1}{1}$ CD $\frac{1}{2}$ DE $\frac{0}{1}$

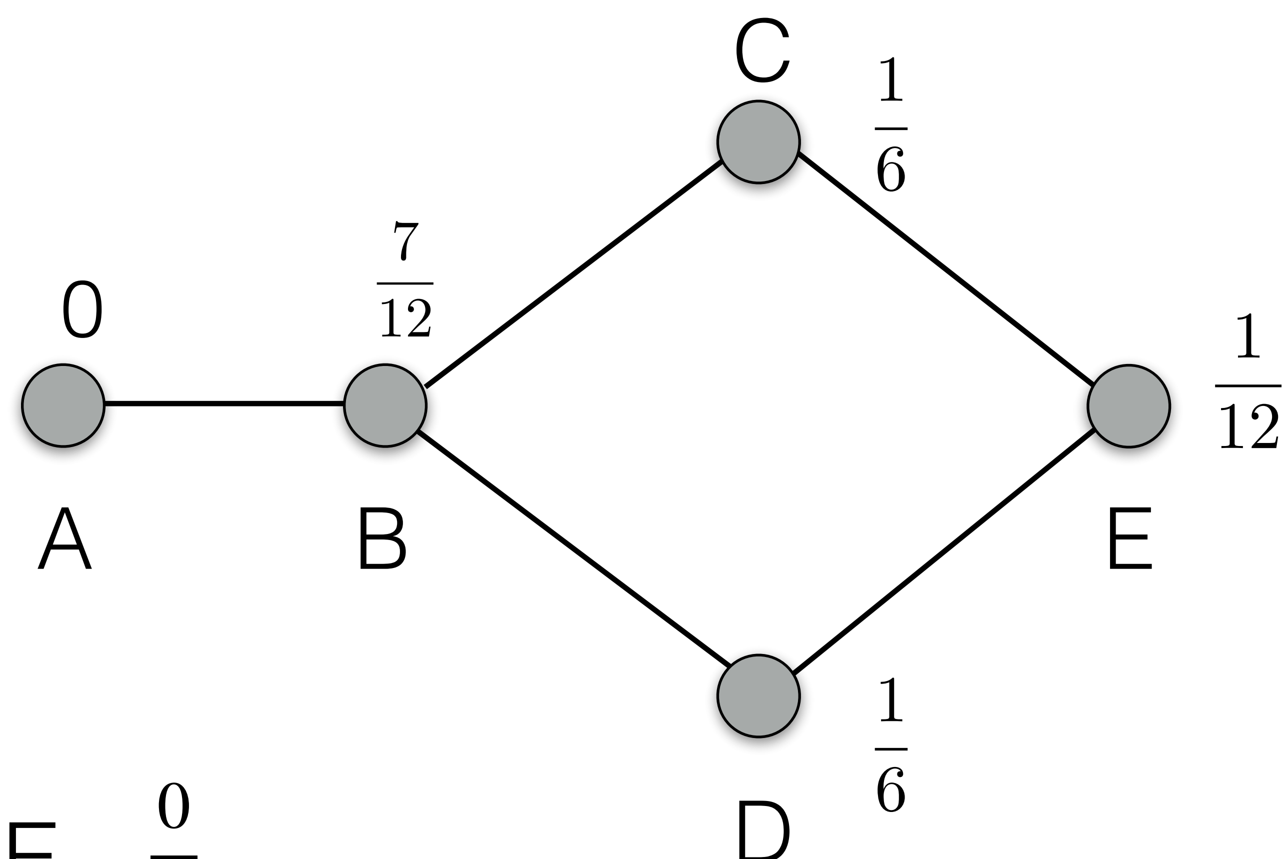
AD $\frac{1}{1}$ CE $\frac{0}{1}$

AE $\frac{1}{1}$

$$\rightarrow C_B(A) = \frac{3(1) + \frac{1}{2} + 2(0)}{6} = \frac{3.5}{6}$$

Betweenness Centrality

$$C_B(v_i) = \frac{\sum_{j<k} \frac{g_{jk}(v_i)}{g_{jk}}}{(N-1)(N-2)/2}$$



AC $\frac{1}{1}$ CD $\frac{1}{2}$ DE $\frac{0}{1}$

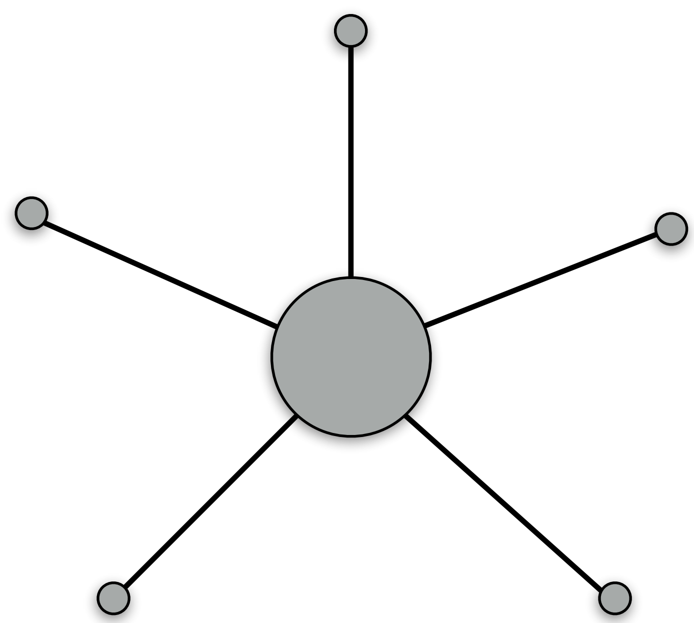
AD $\frac{1}{1}$ CE $\frac{0}{1}$

AE $\frac{1}{1}$

$$\rightarrow C_B(A) = \frac{3(1) + \frac{1}{2} + 2(0)}{6} = \frac{3.5}{6}$$

Betweenness Centrality

- Betweenness centrality make sense when you gain from bridging between different groups



- Brokering between groups
- Control of information
- Innovation
- Collaboration

Eigenvector Centrality

- Fourth notion: you are more important if you're connected to important people
- For example:
  - a small twitter account followed by someone with a large audience
  - a entrepreneur who knows Jack Dorsey
  - an aide to the president
- This is harder to calculate (I would not make you calculate it on an exam)

Eigenvector Centrality

Such a centrality measure must satisfy: $\mathbf{Ax} = \lambda \mathbf{x}$

the sum of the centralities of you neighbors

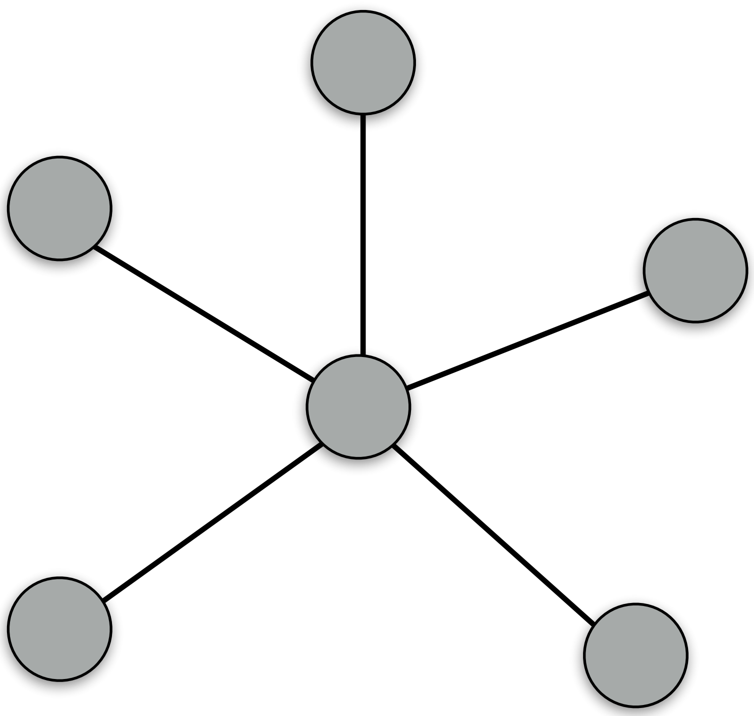
$$C_e\left(v_i\right) = \frac{1}{\lambda_1} \sum_j A_{ij} C_E\left(v_j\right)$$

leading eigenvalue of the matrix A

- A node's eigenvector centrality is proportional to the centrality of it's neighbors
- A node can have higher eigenvector centrality because:
  - They have more connections
  - They have more important connections

Network Centralization

- Centralization: a measure of how centrality is distributed in the network



→An attempt to quantify how centralized the network is as a whole

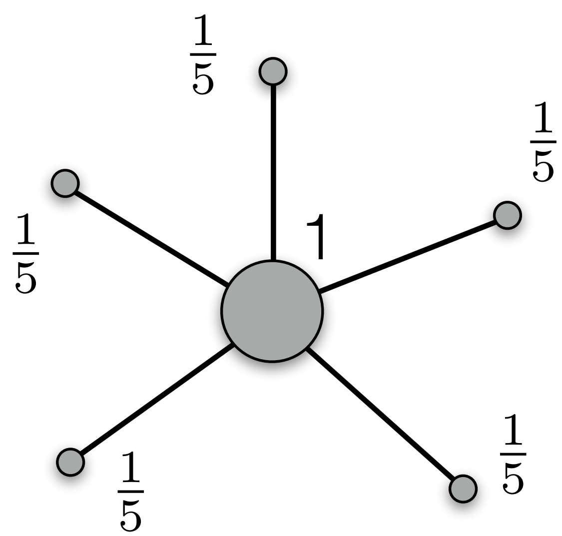
Difference between a node's centrality and the maximum centrality in the network

$$C_D(G) = \frac{\sum_{v_i \in G} [C_D(v^*) - C_D(v_i)]}{(N-1)}$$

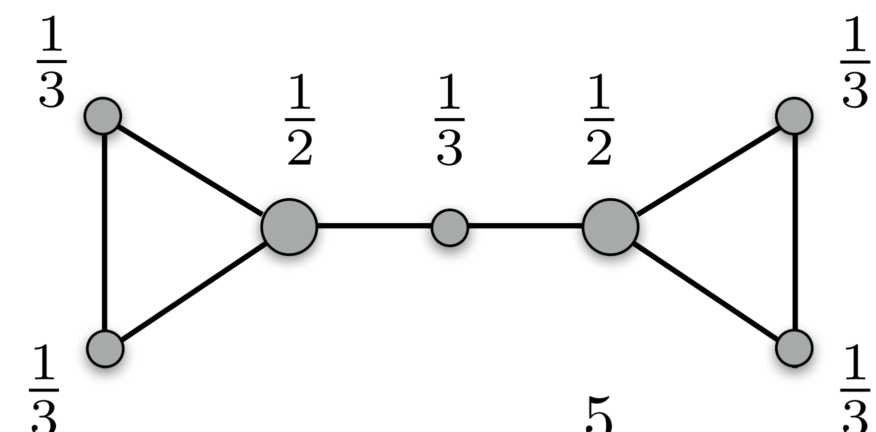
Normalization: if everyone had maximum centrality

Centralization

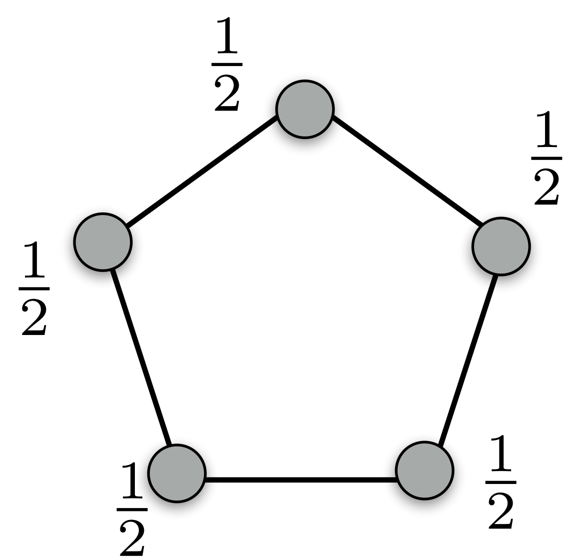
$$C_D(G) = \frac{\sum_{v_i \in G} \left[C_D(v^*) - C_D(v_i) \right]}{(N-1)}$$



$$C_D(G) = \frac{4}{5}$$



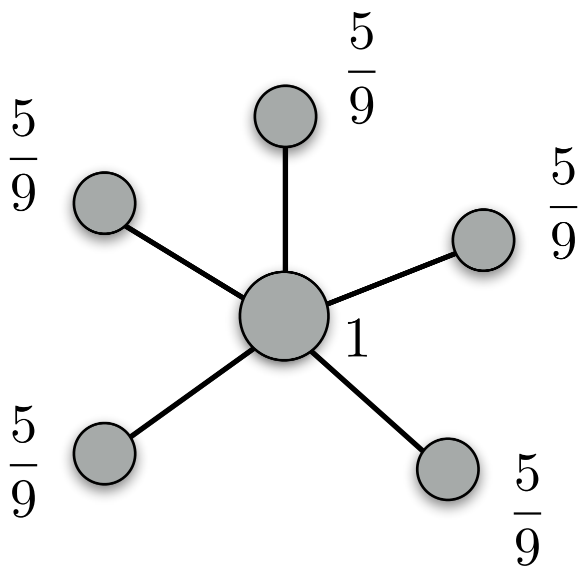
$$C_D(G) = \frac{5}{36}$$



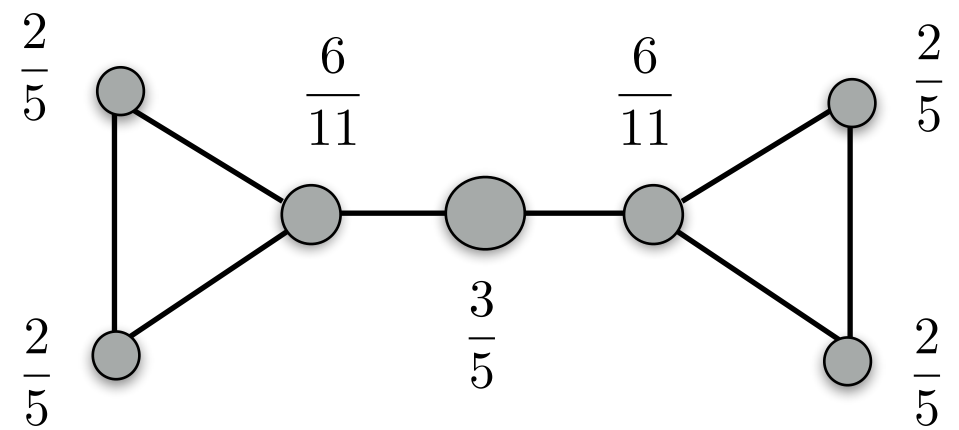
$$C_D(G) = 0$$

Centralization

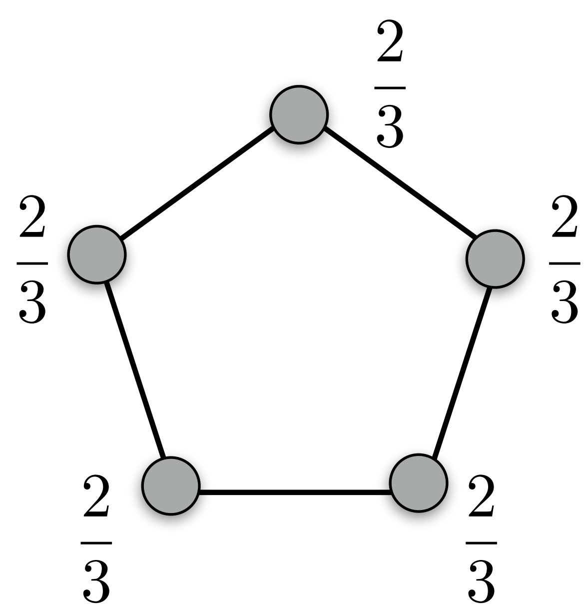
$$C_C(G) = \frac{\sum_{v_i \in G} \left[C_C(v^*) - C_C(v_i)\right]}{(N-1)}$$



$$C_C(G) = \frac{4}{9}$$



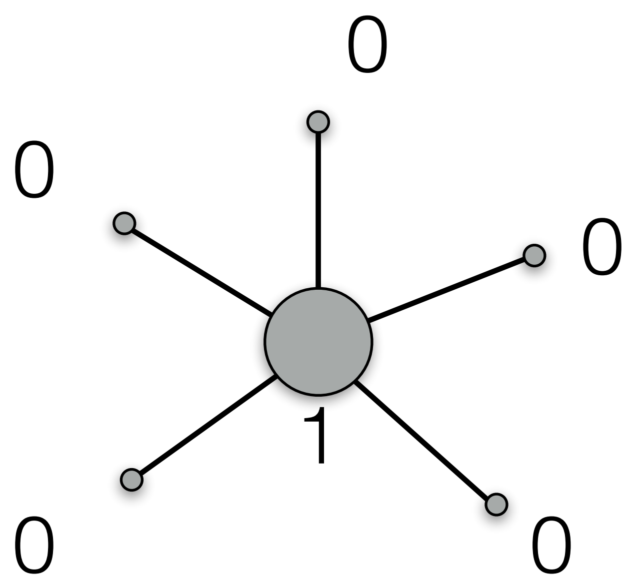
$$C_C(G) = \frac{10}{66}$$



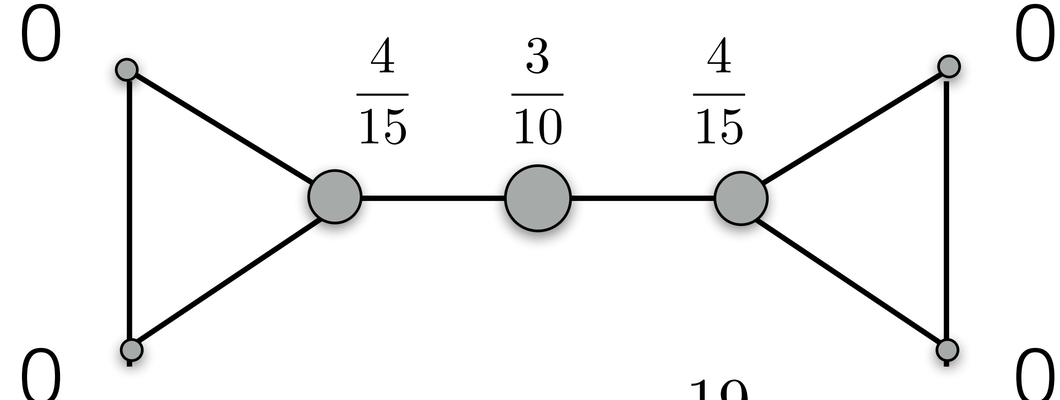
$$C_C(G) = 0$$

Centralization

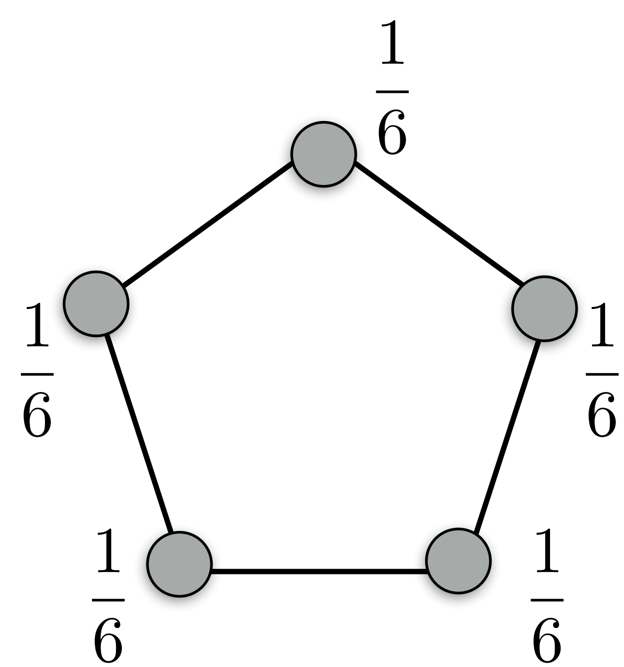
$$C_B(G) = \frac{\sum_{v_i \in G} \left[C_B(v^*) - C_B(v_i) \right]}{(N-1)}$$



$C_B(G) = 1$



$C_B(G) = \frac{19}{90}$

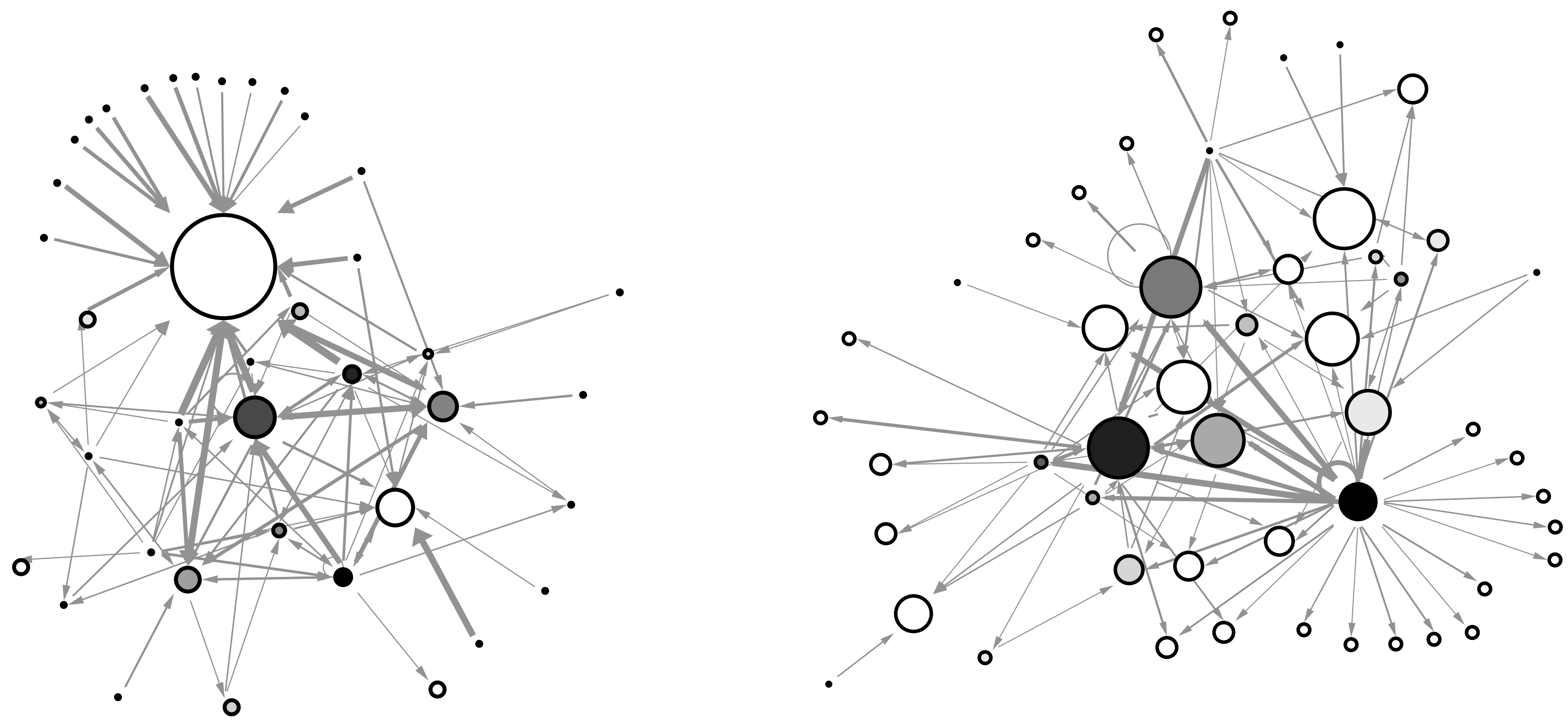


$C_B(G) = 0$

Centralization

Centralization tells us about how influence is spread across the network

Example: Financial Trading Networks

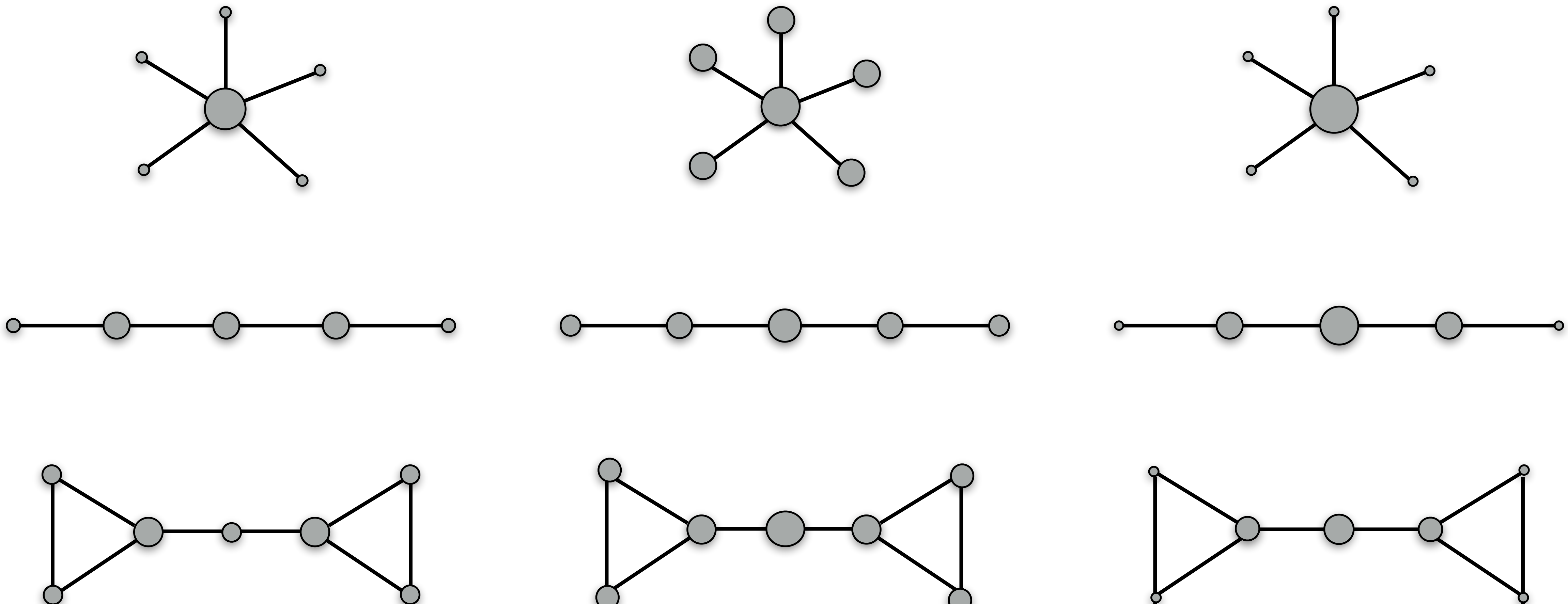


High centralization: one node dominates the network

Low centralization: trades are more evenly distributed

example & graphics: Adamic lecture

Comparing Centrality Measures



Degree Closeness Betweenness

The three are clearly related, but they each get at something slightly different

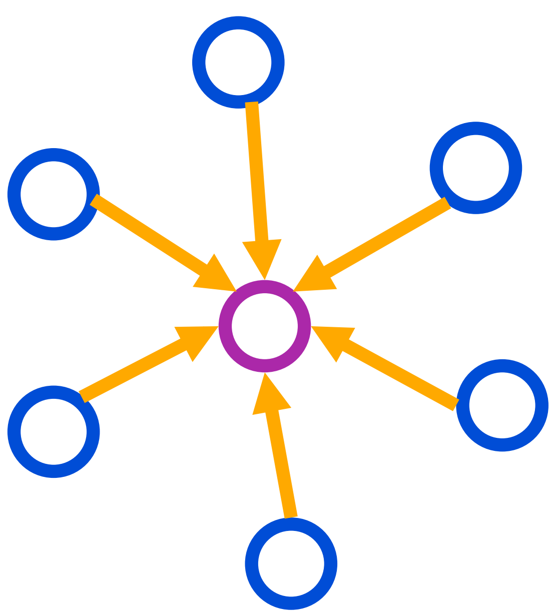
Directed Networks: “Prestige”

- Centrality in directed networks is called “prestige”
- This is sometimes a fine name:
    - admiration or trust
    - influence
    - friendship
    - trade
- But depending on the type of link, it might be misleading:
    - money lending
    - giving advice
    - hatred or distrust

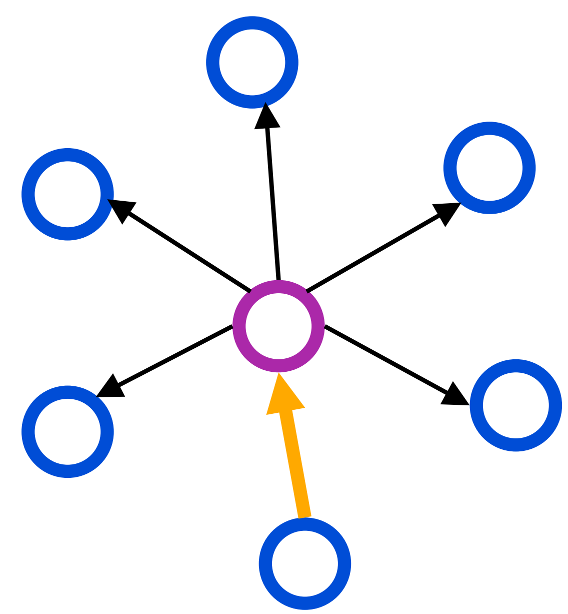
Directed Networks: "Prestige"

- Measure 1: directed version of in-degree
  - A website that is linked to often has high prestige
  - A person who is frequently nominated for a reward has high prestige

$$C_D(v_i) = \frac{d_{in}(v_i)}{(N-1)}$$



High

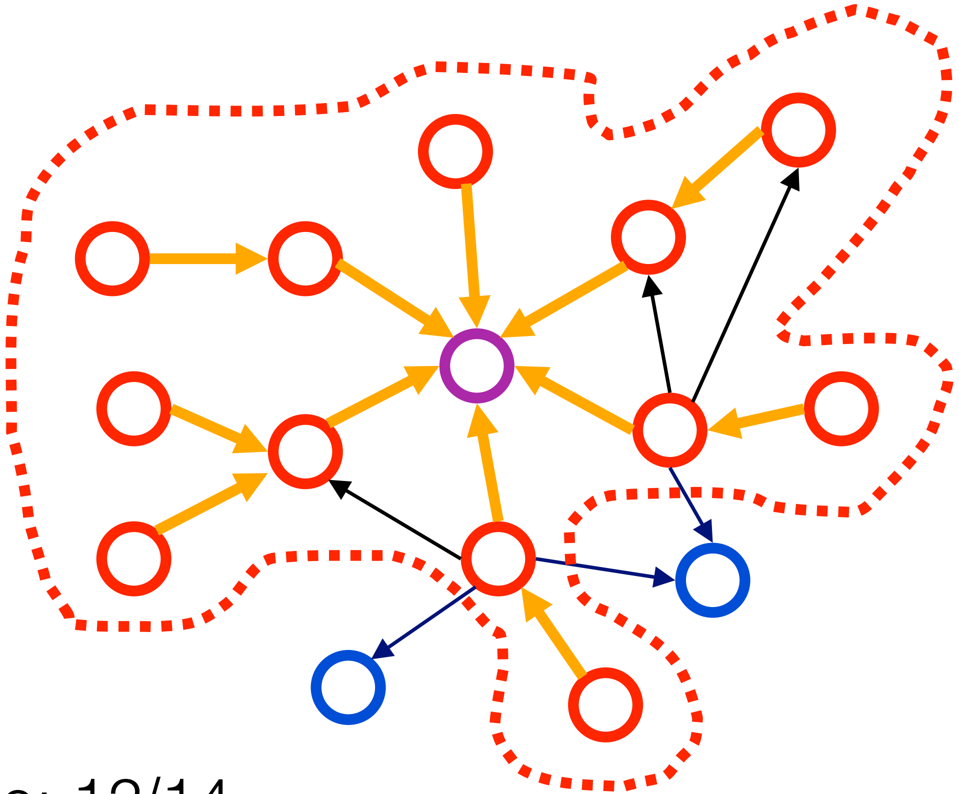


Low

example & graphics: Adamic lecture

Directed Networks: “Prestige”

- Measure 2: Influence range
    - The influence range is what fraction of the nodes in the network can reach you via directed paths



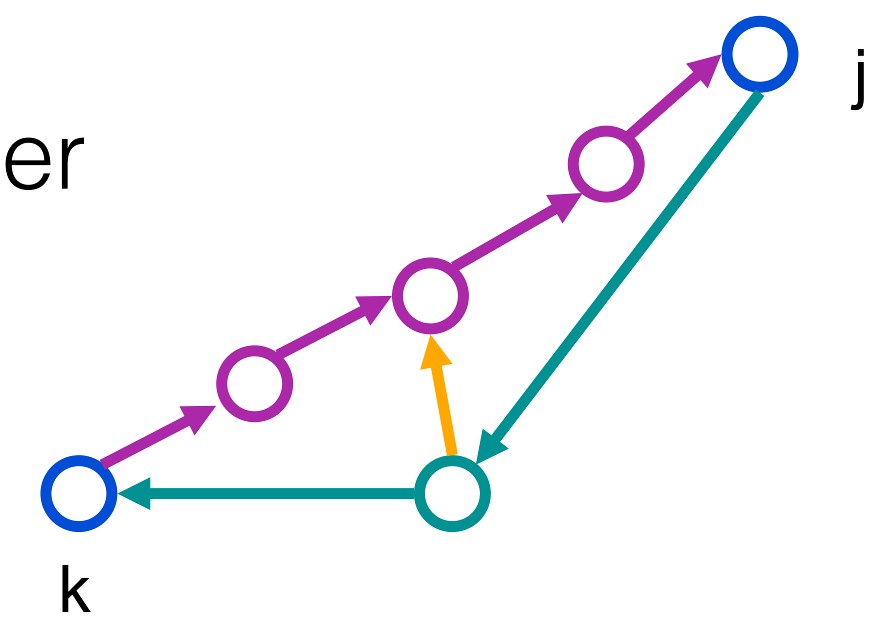
Influence Range: 12/14

example & graphics: Adamic lecture

Directed Networks: "Prestige"

A note on directed geodesics:

- You need to follow the arrows when tracing a path through the network
- The shortest directed path may not be the geodesic on the related undirected network
- The directed geodesic from j to k may be shorter than the directed geodesic from k to j



example & graphics: Adamic lecture

Directed Networks: "Prestige"

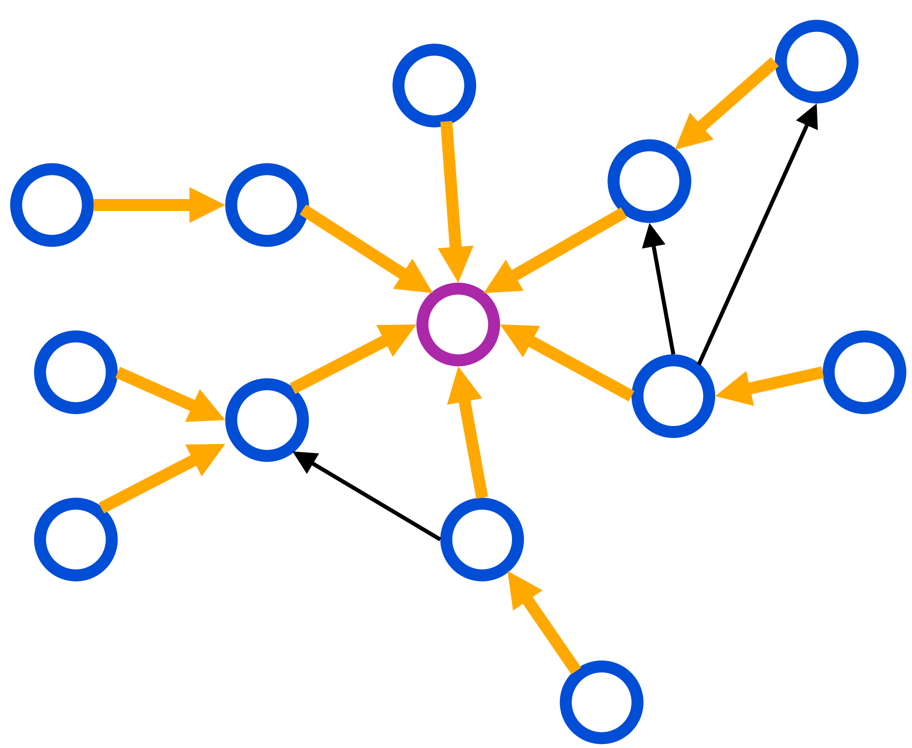
Directed Betweenness: Almost exactly the same as betweenness, but with directed geodesics and normalized in a directed way

$$C_B(v_i) = \frac{1}{(N-1)(N-2)} \sum_{j,k} \frac{g_{jk}(v_i)}{g_{jk}}$$

Number of directed geodesics between j and k containing i

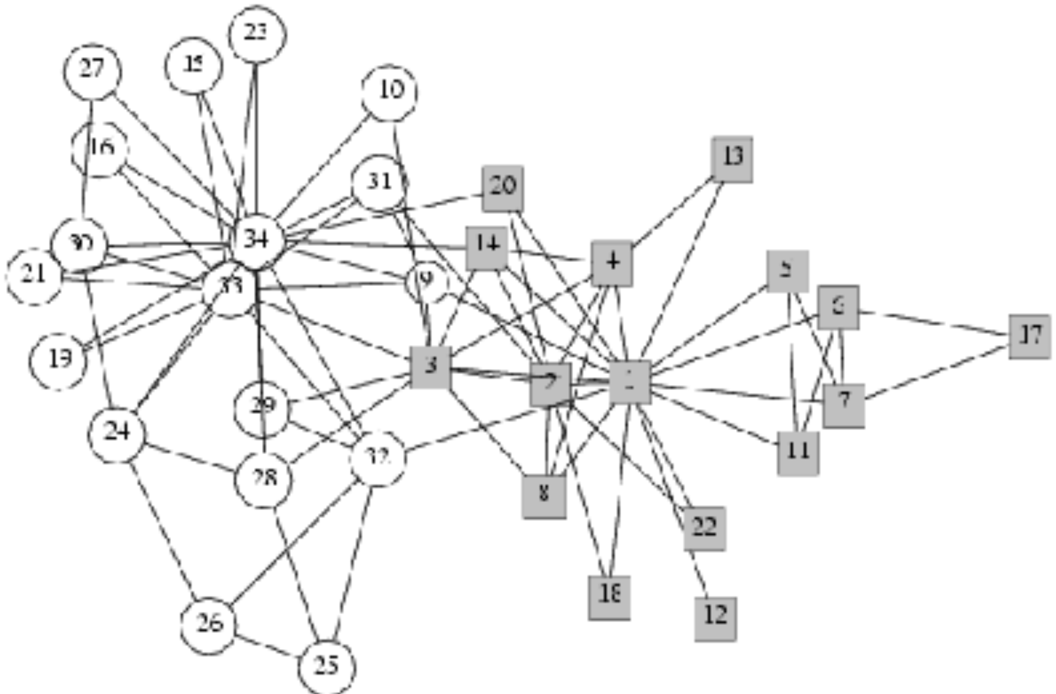
Note: *both* directions

Total number of directed geodesics between j and k



Directed Betweenness Centrality: 0

Summing up…



graphic: Newman Karate Club

There are lots of ways for a node to be "central" to a network

- Degree
- Closeness
- Betweenness
- etc!

- *Different types of centrality will be relevant in different contexts.*
- Which is most interesting is a judgment call!